

Magnetism and Electromagnetic Induction

CHAPTER 6

Alternating Current

6.1–6.28

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6

Alternating Current

INTRODUCTION

We have studied about the dc circuits. But most of the power generated and used in the world is in the form of alternating current (ac). An alternating current is the current whose value changes with time in both magnitude and direction. We will study about a special kind of alternating current which varies sinusoidally. It is represented by $I = I_0 \sin(\omega t + \phi)$

where $I \rightarrow$ instantaneous current

$I_0 \rightarrow$ peak value or maximum value of ac

$\omega t + \phi \rightarrow$ phase at any time t

$\phi \rightarrow$ initial phase or phase constant

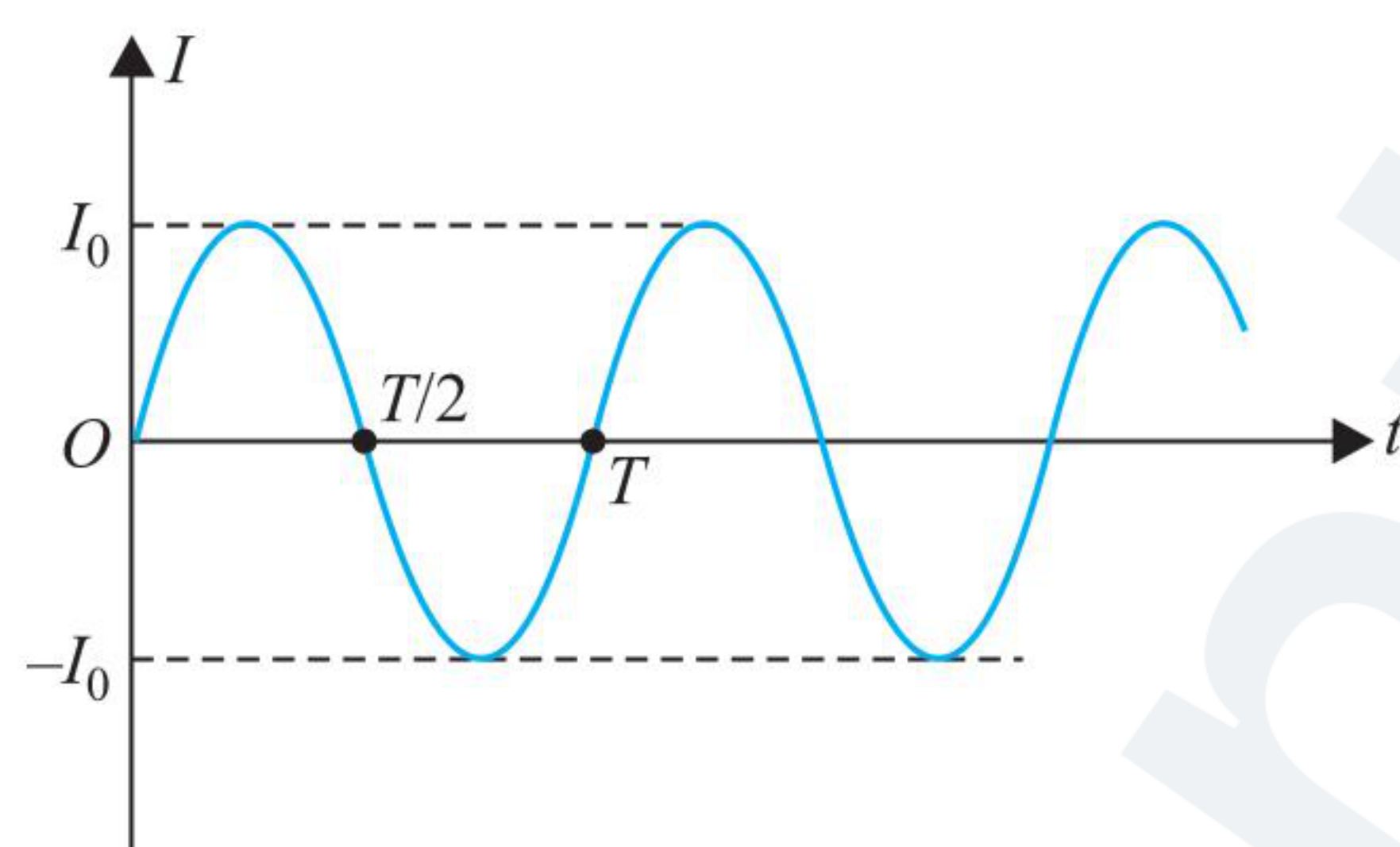
$\omega \rightarrow$ angular frequency

$T \rightarrow$ time period

(where $T = 2\pi/\omega$)

$f \rightarrow$ frequency (where $f = 1/T$)

The graph of I vs. t is shown for $\phi = 0$



Time period, T : It is the time taken to complete one complete cycle or one oscillation (one cycle consists of variation of current from 0 to maximum, maximum to 0, 0 to negative maximum, and negative maximum to 0).

Frequency, f : It is the number of cycles or oscillations completed per unit time.

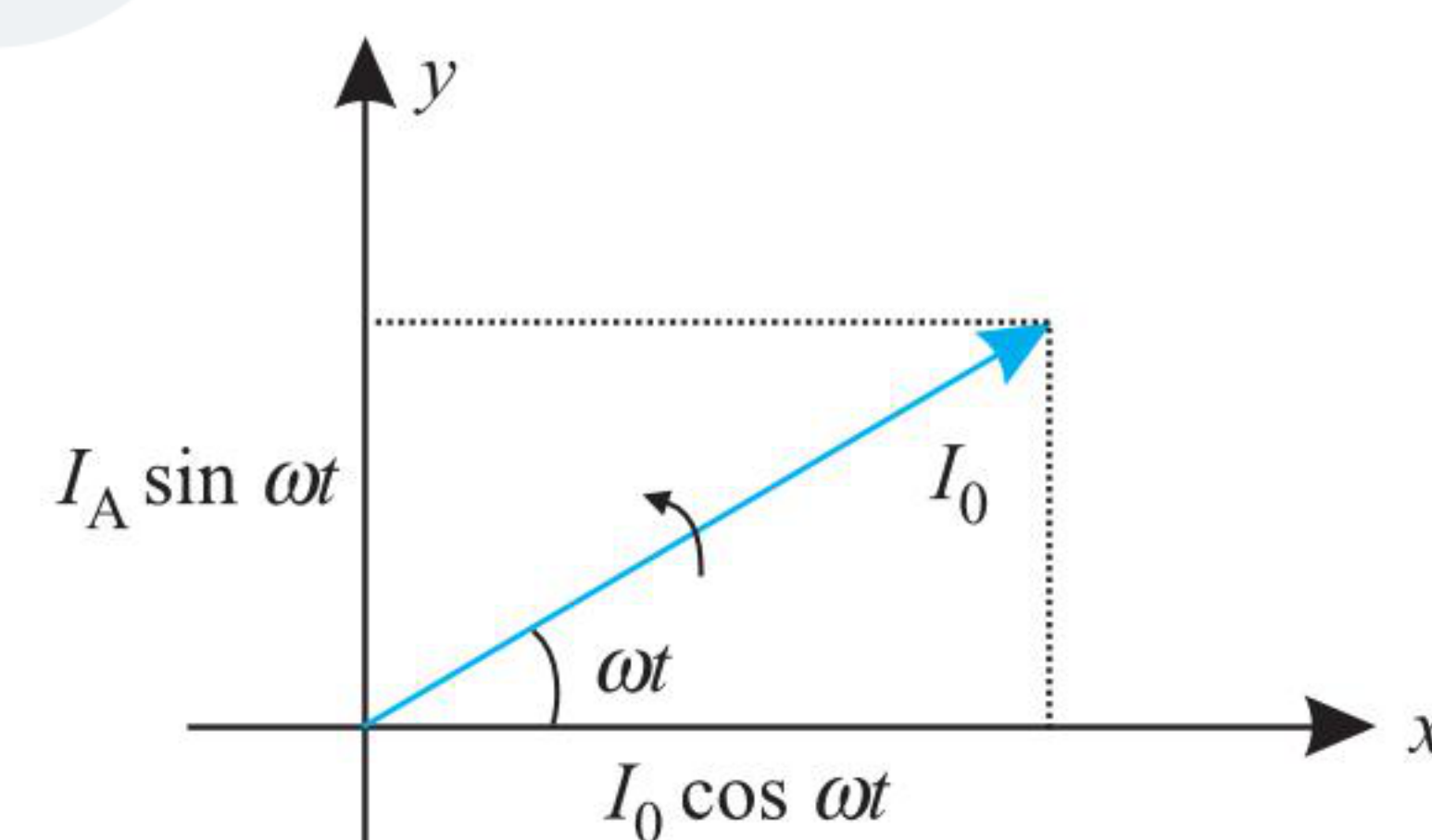
Note:

1. The equation of current can also be written in cosine form as $I = I_0 \cos(\omega t + \phi')$
2. To produce alternating current, emf should be alternating. An alternating emf can be represented by $E = E_0 \sin(\omega t + \phi_1)$
3. An alternating emf is produced by a dynamo or an electronic oscillator.
4. The frequency of ac in India is 50 Hz, i.e., $f = 50$ Hz, so $\omega = 2\pi f = 100\pi$ rad s^{-1} .
5. The ac can be converted into dc with the help of a rectifier while dc into ac with the help of an inverter.
6. It cannot produce chemical effects of current such as electroplating or electrolysis as due to large ions, they cannot follow the frequency of ac.
7. It can be stepped up or stepped down with the help of transformer (while dc cannot be).

PHASOR DIAGRAMS

Generally, currents and voltages in ac circuits are represented in the form of phasors or anticlockwise rotating vectors. The length of arrow represents the peak value of the quantity and its projection on x - or y -axis gives its instantaneous value.

For example, let $I = I_0 \sin \omega t$, then it will be represented as shown in figure. Length of arrow is I_0 which represents the peak value of I . Its projection on y -axis is $I_0 \sin \omega t$ which represents the instantaneous value. ωt is the phase angle which increases with time.



Note: If the equation of current were in cosine form as $I = I_0 \cos \omega t$, then projection on x -axis will represent the instantaneous value.

MEAN OR AVERAGE VALUE OF CURRENT

Mean or Average value of any current in a given time is defined as that constant value of current which will send the same amount of charge in a given circuit as sent by actual current in the same time interval.

$$I_{av}(t_2 - t_1) = \int_{t_1}^{t_2} I(t) dt \Rightarrow I_{av} = \frac{\int_{t_1}^{t_2} I(t) dt}{t_2 - t_1}$$

Average value of $I = I_0 \sin \omega t$

1. Over first half cycle: for this $t_1 = 0, t_2 = T/2 = \pi/\omega$

$$I_{av} \left(\frac{T}{2} - 0 \right) = \int_0^{T/2} I_0 \sin \omega t dt = \left[\frac{-I_0 \cos \omega t}{\omega} \right]_0^{\pi/\omega}$$

$$\Rightarrow I_{av} \left(\frac{T}{2} \right) = \frac{2I_0}{\omega} = \frac{2I_0 T}{2\pi}$$

$$\Rightarrow I_{av} = \frac{2I_0}{\pi} = 0.637 I_0$$

2. Over full cycle: for this $t_1 = 0, t_2 = T = 2\pi/\omega$

$$I_{av}(T - 0) = \int_0^T I_0 \sin \omega t dt = \left[\frac{-I_0 \cos \omega t}{\omega} \right]_0^{2\pi/\omega}$$

$$\Rightarrow I_{av} = 0$$

So average value of current for the full cycle is zero. We can also think in this way:

Average value during positive half cycle: $0.637 I_0$

Average value during negative half cycle: $-0.637 I_0$

So over full cycle, $I_{av} = 0.637 I_0 - 0.637 I_0 = 0$

Note:

1. Average value of emf can also be found in the same way as we find for current. For this we can write the formula:

$$E_{av} = \frac{\int_{t_1}^{t_2} E(t) dt}{t_2 - t_1}$$

2. Average value of alternating current or emf over a long period of time will be zero, because for this time interval $t_2 - t_1$ will be large whereas quantity in the numerator will be finite.

ROOT MEAN SQUARE (RMS) VALUE OF CURRENT

The root mean square value of any current is defined as that value of steady current (constant), which would generate the same amount of heat in a given resistance in a given time as generated by actual current passing through the same resistance for the same given time.

The rms value is also known as *effective value* or *virtual value* of current. It is denoted by I_v .

$$I_v^2 R (t_2 - t_1) = \int_{t_1}^{t_2} [I(t)]^2 R dt \Rightarrow I_v = \sqrt{\frac{\int_{t_1}^{t_2} [I(t)]^2 dt}{t_2 - t_1}}$$

We square the instantaneous current I , take the average (mean) value of I^2 , and finally take the square root of that average. This procedure defines the root mean square current, denoted as I_{rms} or I_v . Even when I is negative, I^2 is always positive, so I_v is never zero (unless I is zero at every instant).

rms value of $I = I_0 \sin \omega t$:

1. Over first half cycle: for this $t_1 = 0, t_2 = T/2 = \pi/\omega$

$$I_v^2 \left(\frac{T}{2} - 0 \right) = \int_0^{T/2} I_0^2 \sin^2 \omega t dt = I_0^2 \int_0^{T/2} \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$\Rightarrow I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

2. Over full cycle: for this $t_1 = 0, t_2 = T = 2\pi/\omega$

$$I_v^2 (T - 0) = \int_0^T I_0^2 \sin^2 \omega t dt = I_0^2 \int_0^T \left(\frac{1 - \cos 2\omega t}{2} \right) dt$$

$$\Rightarrow I_v = \frac{I_0}{\sqrt{2}} = 0.707 I_0$$

The rms value of ac over half cycle or full cycle is same. This is also valid over a long period of time.

Note:

1. Similarly, root mean square value of an emf.

$$E_v = \frac{E_0}{\sqrt{2}} = 0.707 E_0$$

2. Ordinary dc ammeters and voltmeters cannot measure alternating current or alternating emf. They will show zero reading. For this purpose, we use ac ammeters and voltmeters. They are known as hot wire instruments. They measure virtual values of current or voltage. For example, if we measure $I = 10 \sin \omega t$ A from ac ammeter, it will show $I_v = \frac{10}{\sqrt{2}}$ A.

3. The symbol for an ac source is shown below.



ILLUSTRATION 6.1

The electric mains in a house are marked 220 V-50 Hz. Write down the equation for instantaneous voltage.

Sol. Here 220 V is rms voltage. So peak voltage is $220\sqrt{2}$ V. Instantaneous voltage is given by

$$\begin{aligned} E &= E_0 \sin(\omega t + \phi) = 220\sqrt{2} \sin(2\pi 50 t + \phi) \\ &= 220\sqrt{2} \sin(100\pi t + \phi) \end{aligned}$$

ILLUSTRATION 6.2

The plate on the back of a personal computer says that it draws 2.7 A from a 120-V, 60-Hz line. For this computer, what is (a) the average of the square of the current, (b) the current amplitude, (c) the average current for a positive half cycle, and (d) the average current for a full cycle?

Sol.

- (a) The current given is the rms value: $I_{rms} = 2.7$ A. The average of the square of the current is nothing but $(I_{rms})^2$.

$$\text{So, } (I_{rms})^2 = (2.7 \text{ A})^2 = 7.3 \text{ A}^2$$

- (b) The current amplitude I_0 is

$$I_0 = \sqrt{2} I_{rms} = \sqrt{2} (2.7 \text{ A}) = 3.8 \text{ A}$$

- (c) For a positive half cycle, $I_{av} = \frac{2I_0}{\pi} = \frac{2 \times 3.8}{\pi} = 2.42 \text{ A}$

- (d) For a full cycle, $I_{av} = 0$

ILLUSTRATION 6.3

An alternating current is given by the following equation: $I = 3\sqrt{2} \sin(100\pi t + \pi/4)$. Give the frequency and rms value of the current.

Sol. $I = 3\sqrt{2} \sin[100\pi t + (\pi/4)]$

We have $2\pi f = 100\pi \Rightarrow f = 50 \text{ Hz}$

$$\text{and } I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}} \text{ A} = 3 \text{ A}$$

ILLUSTRATION 6.4

A voltage, $E = 60 \sin(314t)$, is applied across a resistor of 20Ω . What will be the reading of I_{rms}

- (a) in an ac ammeter?

- (b) in an ordinary moving coil ammeter in series with the resistor?

Sol. Given that $E = 60 \sin(314t)$

(a) An ac ammeter will read the rms value

$$E_{\text{rms}} = \frac{E_0}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.4 \text{ V}$$

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{R} = \frac{42.4}{20} = 2.12 \text{ A}$$

Therefore, ac ammeter will read 2.12 A.

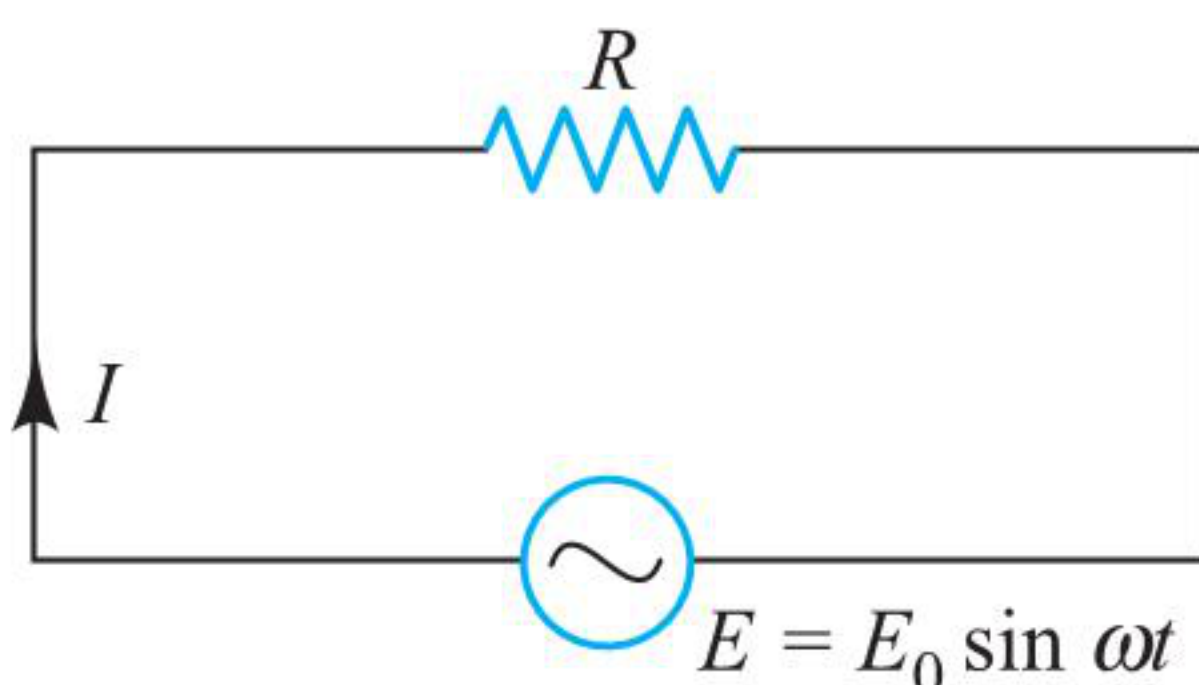
(b) An ordinary moving coil ammeter will read the average value of alternating current. Since the average value of ac is zero, this meter will give zero reading.

AC CIRCUITS

Basic ac circuit elements are resistors, inductors, and capacitors. We will discuss the behaviour of each of them when connected in ac circuits.

AC CIRCUITS CONTAINING RESISTOR ONLY

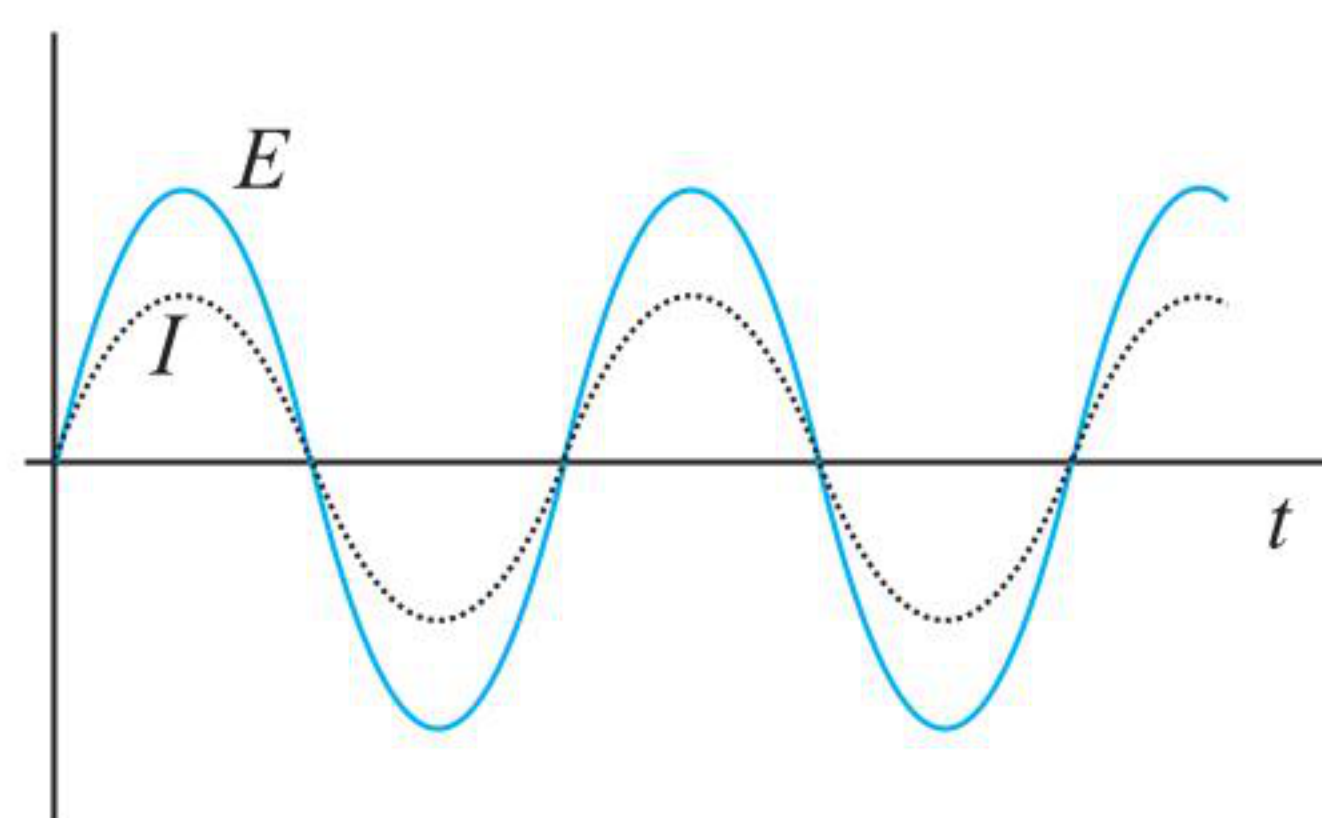
Consider resistor R is connected across emf $E = E_0 \sin \omega t$ as shown in figure. Due to this emf, current will flow in the resistance which will vary in both magnitude and direction. At any time, current is given by



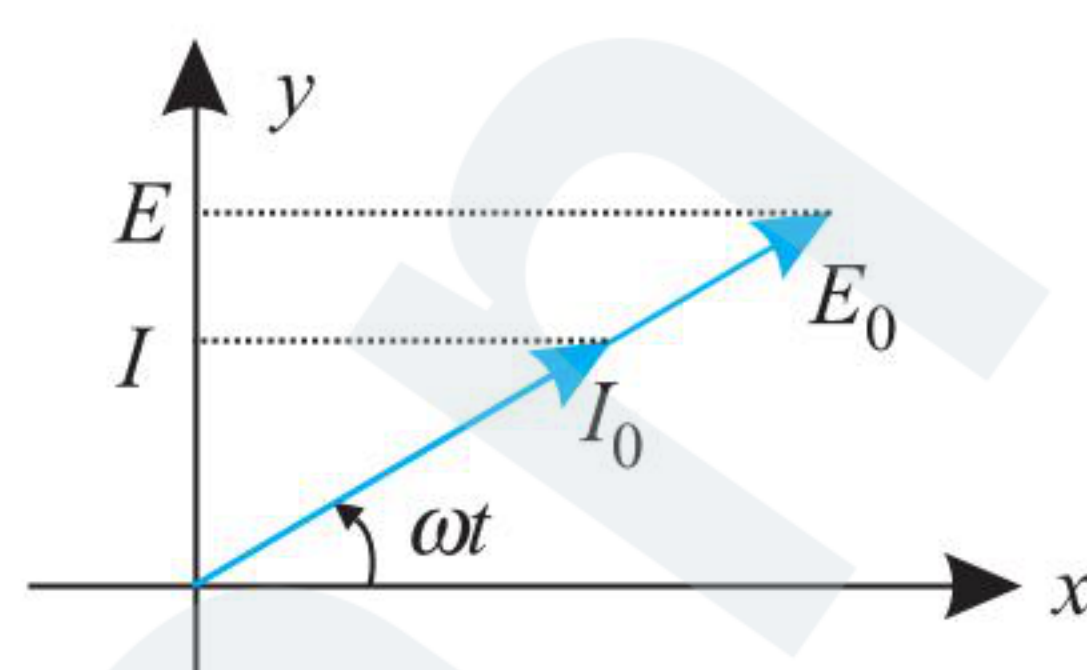
$$I = \frac{E}{R} = \frac{E_0}{R} \sin \omega t$$

$$= I_0 \sin \omega t \quad [\text{where } I_0 = \frac{E_0}{R} \rightarrow \text{Current amplitude}]$$

Variation of E and I is shown in Fig. (a) and their phasor diagram representation is shown in Fig. (b).



(a)



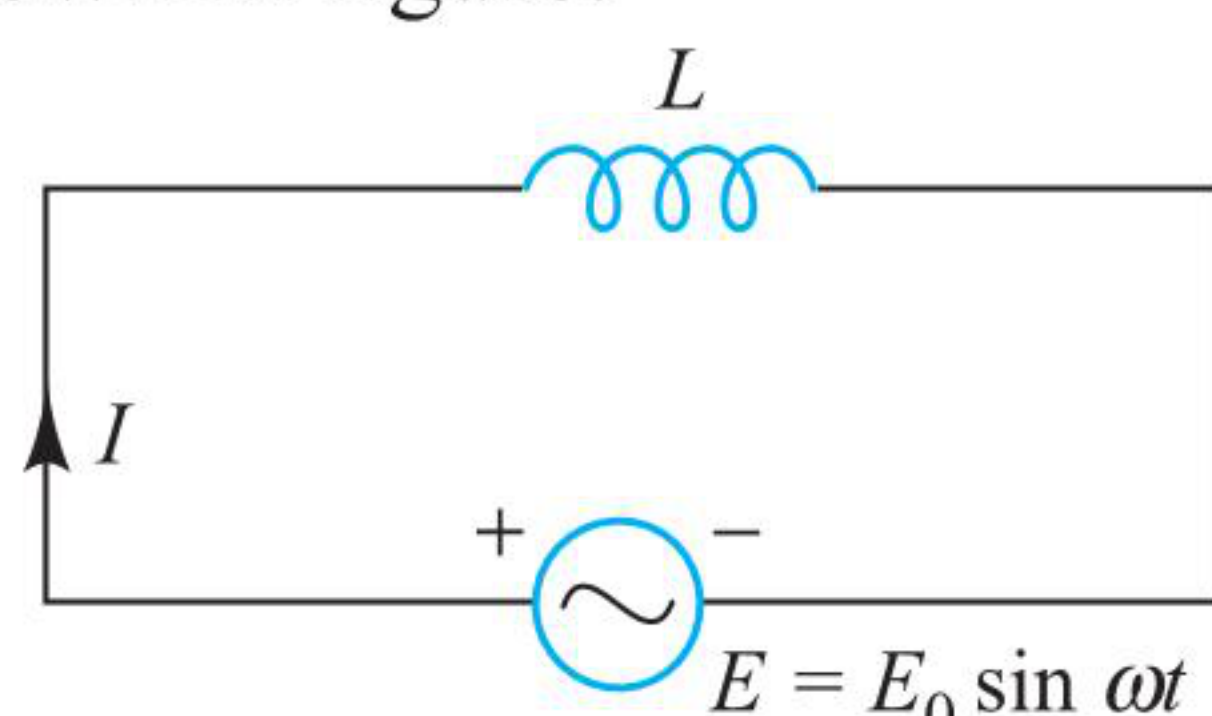
(b)

From both the diagrams we see that phase of current and emf are same.

Same phase means both are varying in the same manner. Both are becoming zero at the same time, attaining their positive maximum, and negative maximum values at the same time, etc. The current and voltage phasors rotate together; they are parallel at each instant. Their projections on the vertical axis represent their instantaneous values.

AC CIRCUITS CONTAINING INDUCTOR ONLY

Consider a pure inductor (having no resistance) of inductance L is connected as shown in figure.



Induced emf in the inductor (with the polarities as shown in the figure, assuming current increases with time): $e = L \frac{dI}{dt}$

As there is only inductor in the circuit, induced emf should be equal to applied emf.

$$\Rightarrow \frac{L dI}{dt} = E_0 \sin \omega t$$

$$\Rightarrow \int dI = \int \frac{E_0}{L} \sin \omega t dt$$

$$\Rightarrow I = -\frac{E_0}{\omega L} \cos \omega t + C$$

where C is constant of integration. To determine the value of C , let us take the average of current for one time period.

$$I_{\text{av}} = \frac{1}{T} \int_0^T I dt = \frac{1}{T} \int_0^T \left(-\frac{E_0}{\omega L} \cos \omega t + C \right) dt$$

$$= \frac{1}{T} \left[-\frac{E_0}{\omega^2 L} \sin \omega t + Ct \right]_0^T = \frac{1}{T} [Ct] = C$$

But for any kind of alternating current $I_{\text{av}} = 0$ over one cycle. Hence, $C = 0$. So, finally, we get

$$I = -\frac{E_0}{\omega L} \cos \omega t$$

$$= \frac{E_0}{\omega L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

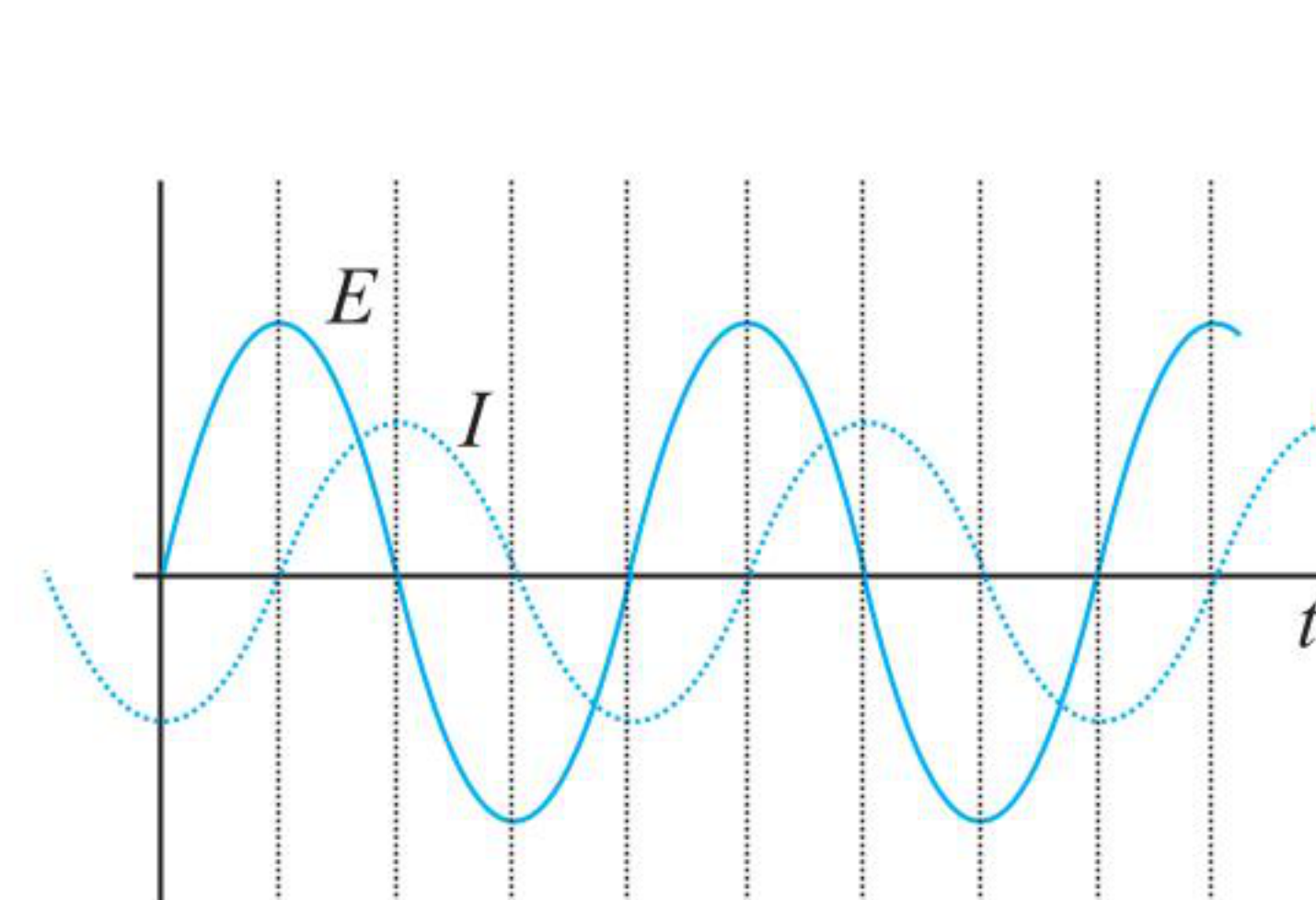
$$= \frac{E_0}{X_L} \sin \left(\omega t - \frac{\pi}{2} \right)$$

$$= I_0 \sin \left(\omega t - \frac{\pi}{2} \right)$$

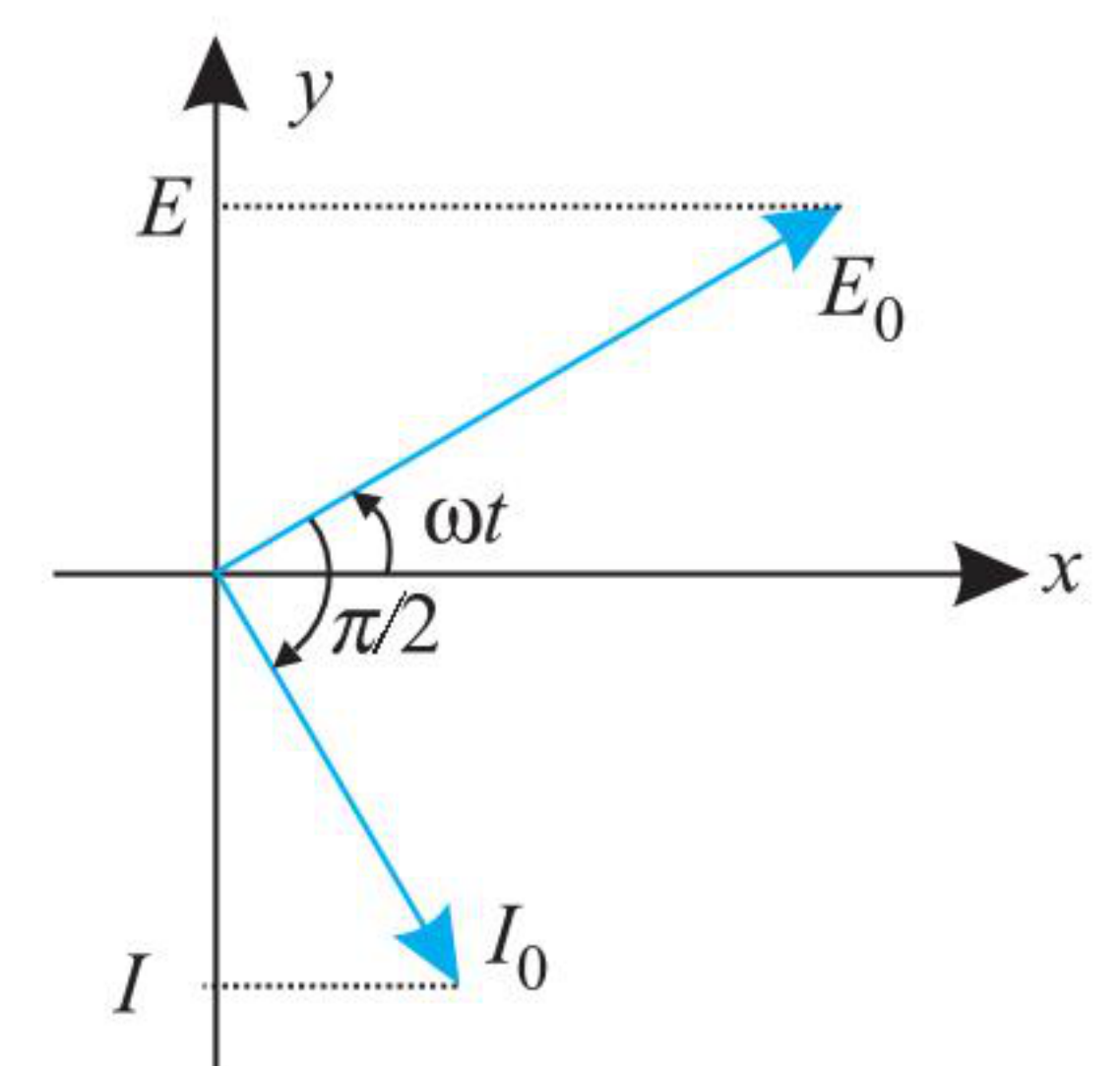
where $X_L = \omega L$ is the resistance offered by inductor. It is also known as inductive reactance. Its unit is Ω .

$$I_0 = \frac{E_0}{X_L} \rightarrow \text{Current amplitude}$$

Variation of E and I is shown in Fig. (a) and their phasor diagram representation is shown in Fig. (b).



(a)



(b)

From Fig. (a), voltage and current are “out of step” or out of phase by a quarter cycle. Voltage peak occurs a quarter cycle earlier than the current peak. Thus, we say as that the voltage leads the current by 90° .

Also from Fig. (b), we see that the phase of emf is ωt and that of current is $\omega t - \pi/2$. So, phase of current is $\pi/2$ less than emf. Hence, current lags behind emf by phase $\pi/2$, or emf leads current by phase $\pi/2$.

A phase difference of $\pi/2$ is equivalent to time difference of $T/4$. We see that current attains its zero or maximum (positive or negative) values after time $T/4$ of emf has attained the same type of value.

Note:

1. For dc circuit $\omega = 0 \Rightarrow X_L = 0$. So inductor offers zero resistance to a dc circuit.
2. X_L depends upon the frequency of source.

INDUCTIVE REACTANCE

Inductive reactance X_L is a description of the self-induced emf that opposes any change in the current through the inductor. Inductive reactance and self-induced emf increase with more rapid variation in current (that is, increasing angular frequency, ω) and increasing inductance L .

If an oscillating voltage of a given amplitude E_0 is applied across the inductor terminals, the resulting current will have a smaller amplitude I for larger values of X_L , since X_L is proportional to frequency, a high-frequency voltage applied to the inductor gives only a small current, while a lower-frequency voltage of the same amplitude gives rise to a larger current. Inductors are used in some circuit applications, such as power supplies and radio-interference filters, to block high frequencies while permitting lower frequencies or dc to pass through. A circuit device that uses an inductor for this purpose is called a low-pass filter.

ILLUSTRATION 6.5

Suppose you want the current amplitude in a pure inductor in a radio receiver to be $250 \mu\text{A}$ when the voltage amplitude is 3.60 V at a frequency of 1.60 MHz (corresponding to the upper is 3.60 V AM broadcast band). What inductive reactance is needed? What inductance is required?

Sol. We are given the current amplitude, I_0 , and the voltage amplitude, E_0 . The inductive reactance

$$X_L = \frac{E_0}{I_0} = \frac{3.60}{250 \times 10^{-6}} = 1.44 \times 10^4 \Omega = 14.4 \text{ k}\Omega$$

$$L = \frac{X_L}{2\pi f} = \frac{1.44 \times 10^4 \Omega}{2\pi(1.60 \times 10^6 \text{ Hz})} = 1.43 \times 10^{-3} \text{ H} = 1.43 \text{ mH}$$

ILLUSTRATION 6.6

A pure inductance of 1.0 H is connected across a 110 V , 70 Hz source. Find the (a) reactance, (b) current, and (c) peak value of current.

Sol. $L = 1.0 \text{ H}$, $V = 110 \text{ V}$, $f = 70 \text{ Hz}$

(a) Reactance $= \omega L = 2\pi \times 70 \times 1 = 440 \Omega$

(b) Current $= 110/440 = 0.25 \text{ A}$

(c) Peak value of current $= 0.25 \sqrt{2} = 0.353 \text{ A}$

AC CIRCUITS CONTAINING CAPACITOR ONLY

Consider a capacitor of capacitance C is connected as shown in figure. Charge on the capacitor at any time is

$$q = CE = CE_0 \sin \omega t$$

Current in the circuit at any time is

$$I = \frac{dq}{dt} = CE_0 \omega \cos \omega t$$

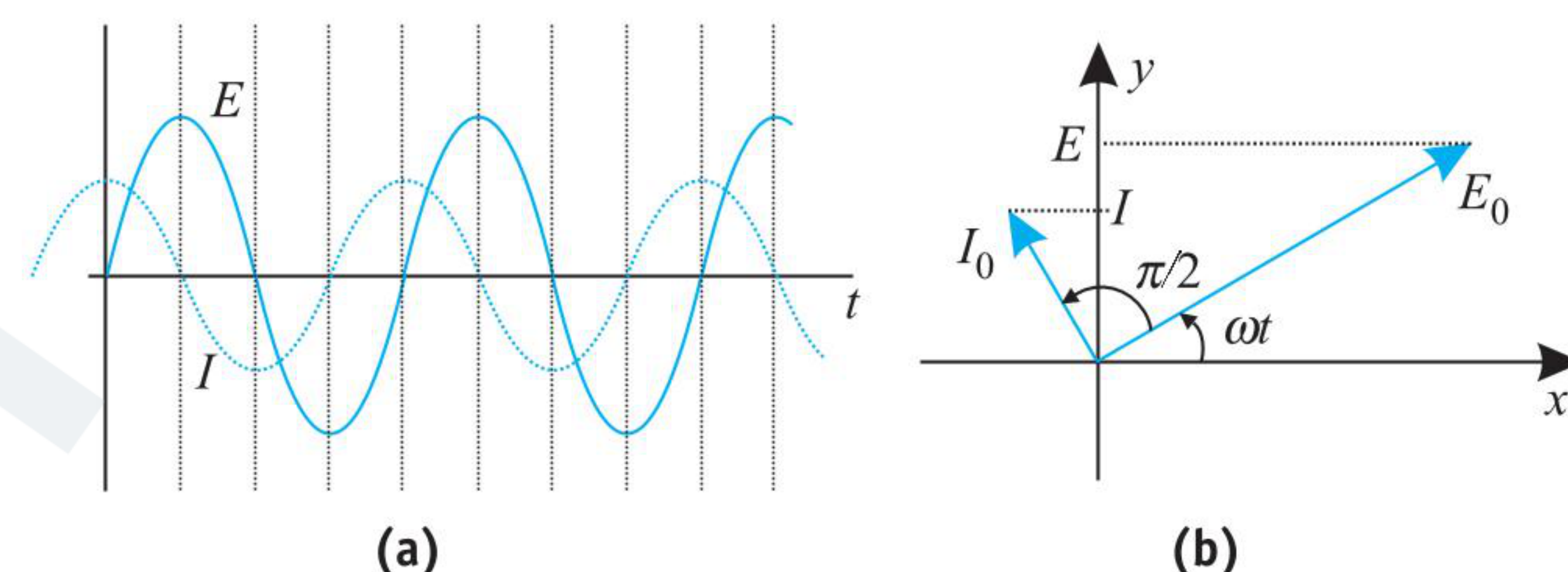
$$\Rightarrow I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\Rightarrow I = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$\text{where } I_0 = \frac{E_0}{X_C} \text{ and } X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

X_C represents effective resistance offered by the capacitor. It is known as capacitive reactance. More the frequency, lesser is the reactance. Further, if $f = 0$, $X_C = \infty$. So for dc circuit, capacitor offers infinite resistance or it blocks dc. Unit of X_C is Ω .

Variation of E and I is shown in Fig. (a) and their phasor diagram representation is shown in Fig. (b).



The capacitor voltage and the current are out of phase by a quarter cycle. The peak of voltage occurs quarter cycle after the corresponding current peak. And we say that the voltage phasor is behind the current phasor by a quarter cycle or 90° .

CAPACITIVE REACTANCE

The capacitive reactance of a capacitor is inversely proportional to both the capacitance C and the angular frequency ω . Greater the capacitance and higher the frequency, smaller is the capacitive reactance, X_C . Capacitors tend to pass high-frequency current and block low-frequency currents and dc, just the opposite of inductors. A device that preferentially passes signals of high frequency is called a high-pass filter.

Note: In case of resistor, resistance is offered due to the obstruction to the passage of current. In inductor, it is induced emf. In capacitor, it is potential difference developed across the capacitor.

The whole of the above discussion can be generalized as:

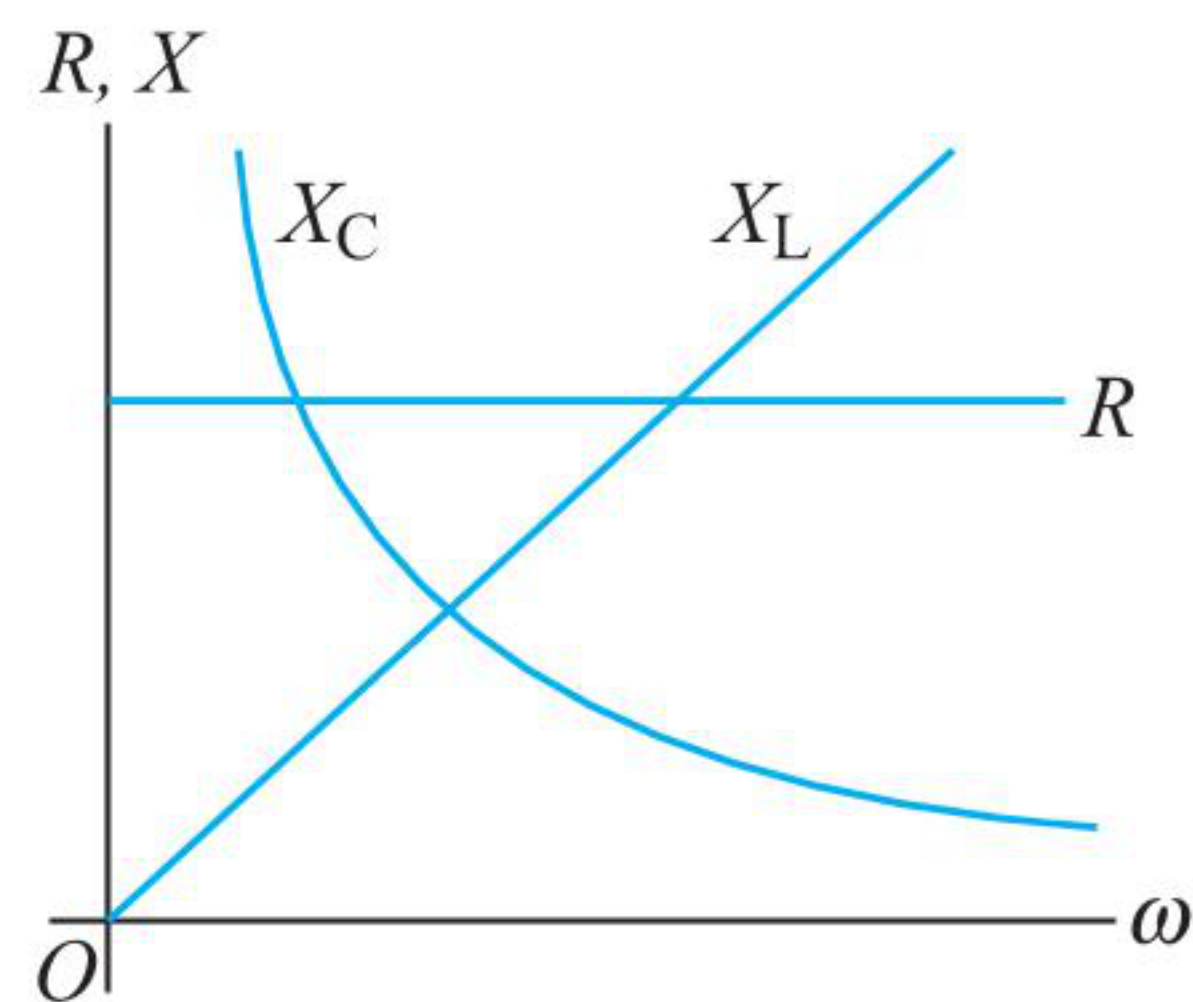
Let $E = E_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$. Here we have assumed that current leads emf by an angle, ϕ . ϕ is also known as phase factor. Here $I_0 = E_0/Z$, where Z is known as impedance of the circuit. Its unit is Ω . We can also write $Z = \frac{E_0}{I_0} = \frac{E_v}{I_v}$

Values of ϕ and Z for different types of circuits is given in the table below:

Type of circuit	Phase factor	Impedance
Purely resistive circuit	$\phi = 0^\circ$	$Z = R$
Purely inductive circuit	$\phi = -\pi/2$	$Z = X_L = \omega L$
Purely capacitive circuit	$\phi = \pi/2$	$Z = X_C = \frac{1}{\omega C}$

The reciprocal of reactance is called susceptance and that of impedance is called admittance.

Figure shows how the resistance of a resistor and the reactances of an inductor and a capacitor vary with angular frequency, ω . Resistance R is independent of frequency, while the reactances X_L and X_C are not. If $\omega = 0$, corresponding to a dc circuit, there is no current through a capacitor because $X_C \rightarrow \infty$, and there is no inductive effect because $X_L = 0$. In the limit $\omega \rightarrow \infty$, X_L also approaches infinity, and the current through an inductor becomes vanishingly small; recall that the self-induced emf opposes rapid changes in the current. In this same limit, X_C and the voltage across a capacitor both approach zero; the current changes direction so rapidly that no charge can build up on either plate.

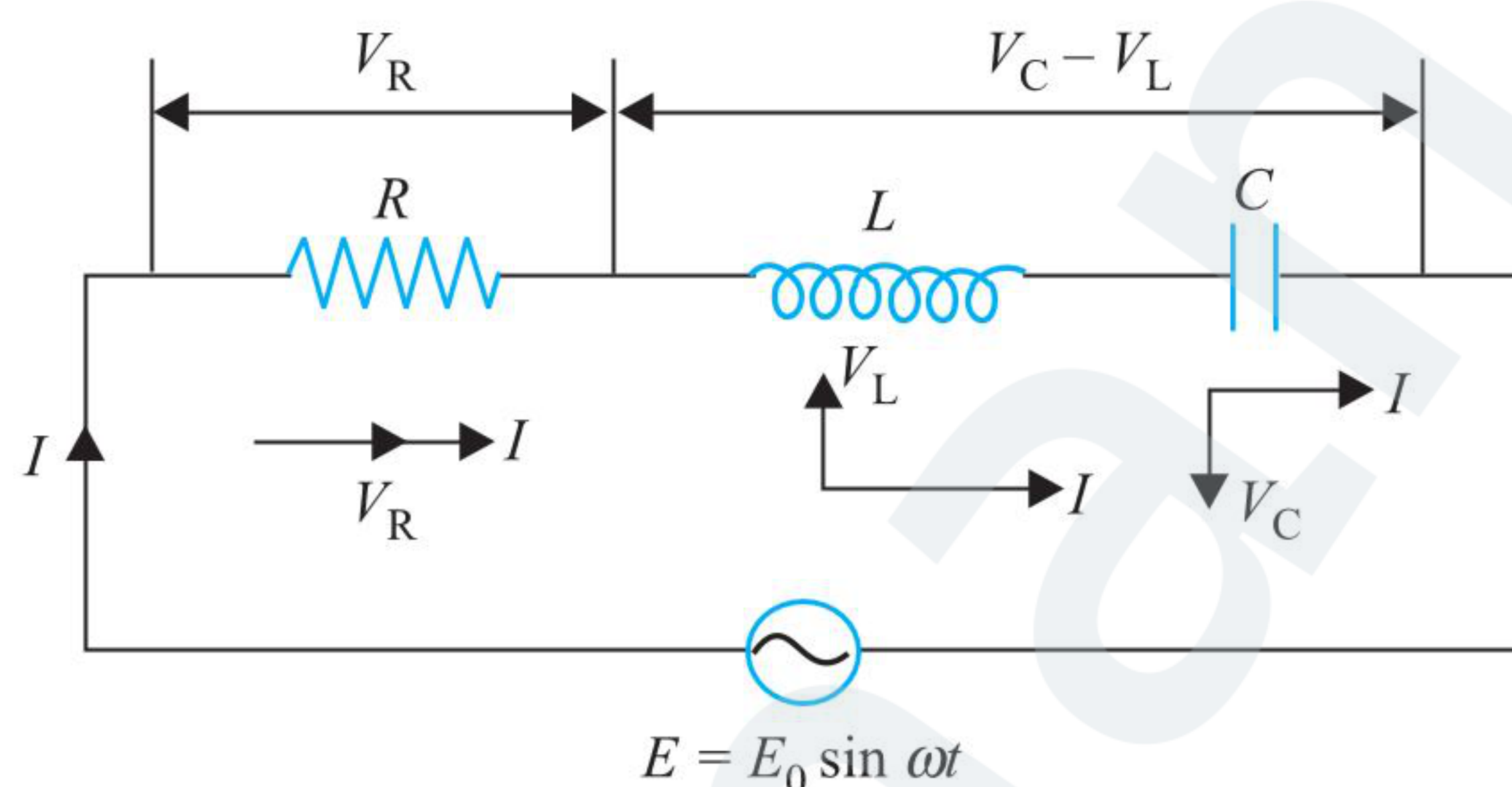


AC CIRCUIT CONTAINING RESISTOR, INDUCTOR, AND CAPACITOR IN SERIES (SERIES LCR CIRCUIT)

Figure shows a circuit in which a resistor of resistance R , an inductor of inductance L , and a capacitor of capacitance C are connected in series across an alternating source of emf

$$E = E_0 \sin \omega t \quad \dots(i)$$

We want to determine the instantaneous current I in the circuit and its phase relationship to the applied voltage.



Let us assume that current I is flowing in the circuit as

$$I = I_0 \sin(\omega t + \phi) \quad \dots(ii)$$

where I_0 is peak current and ϕ is the phase by which current leads emf.

V_R , V_L , and V_C are the instantaneous voltages across R , L and C , respectively. We can write the following relations:

$$V_R = V_{R0} \sin(\omega t + \phi) \quad \dots(iii)$$

$$\begin{aligned} V_L &= V_{L0} \sin\left(\omega t + \phi + \frac{\pi}{2}\right) \\ &= V_{L0} \cos(\omega t + \phi) \end{aligned} \quad \dots(iv)$$

$$\begin{aligned} V_C &= V_{C0} \sin\left(\omega t + \phi - \frac{\pi}{2}\right) \\ &= -V_{C0} \cos(\omega t + \phi) \end{aligned} \quad \dots(v)$$

The above equations are written keeping in mind the phase relationship between current and voltage across each of the element resistor, inductor, and capacitor.

Equation (iii) shows that current and voltage across R are in phase (having same phase $\omega t + \phi$).

Equation (iv) shows that voltage across inductor leads the current by $\pi/2$.

Equation (v) shows that voltage across capacitor lags current by $\pi/2$.

V_{R0} , V_{L0} , and V_{C0} are the peak values of voltages across R , L , and C , respectively. We can write

$$\begin{cases} V_{R0} = I_0 R \\ V_{L0} = I_0 X_L \\ V_{C0} = I_0 X_C \end{cases} \quad \dots(vi)$$

At any instant:

$$\begin{aligned} E &= V_R + V_L + V_C \\ \Rightarrow E_0 \sin \omega t &= V_{R0} \sin(\omega t + \phi) \\ &\quad + (V_{L0} - V_{C0}) \cos(\omega t + \phi) \end{aligned} \quad \dots(vii)$$

Equation (vii) is a general equation and it is true at any instant of time. Replacing ωt by $\omega t + \pi/2$ in Eq. (vii), we get

$$E_0 \cos \omega t = V_{R0} \cos(\omega t + \phi) - (V_{L0} - V_{C0}) \sin(\omega t + \phi) \quad \dots(viii)$$

Squaring and adding (vii) and (viii),

$$\begin{aligned} E_0^2 &= V_{R0}^2 + (V_{L0} - V_{C0})^2 \quad \dots(ix) \\ \Rightarrow &= I_0^2 [R^2 + (X_L - X_C)^2] \quad [\text{using (vi)}] \\ \Rightarrow \frac{E_0}{I_0} &= \sqrt{R^2 + (X_L - X_C)^2} \\ \Rightarrow Z &= \sqrt{R^2 + (X_L - X_C)^2} \quad \dots(x) \end{aligned}$$

Equation (x) gives impedance of the circuit.

Dividing Eq. (vii) by Eq. (viii) and simplifying, we get

$$\begin{aligned} \tan \phi &= \frac{V_{C0} - V_{L0}}{V_{R0}} \quad \dots(xi) \\ \Rightarrow &= \frac{I_0 X_C - I_0 X_L}{I_0 R} \\ \Rightarrow &= \frac{X_C - X_L}{R} \quad \dots(xii) \end{aligned}$$

Using Eq. (xii), we can find ϕ , the phase difference between current and voltage. It is to be noted that $-\pi/2 \leq \phi \leq \pi/2$.

If $X_C < X_L$, then ϕ is negative

If $X_C > X_L$, then ϕ is positive

If $X_C = X_L$, then $\phi = 0$

If $R = 0$, then $\phi = \frac{\pi}{2}$ (if $X_C > X_L$)

or $\phi = -\frac{\pi}{2}$ (if $X_L > X_C$)

Note:

Equation (ix) can also be written in terms of virtual voltages.

$$E_v^2 = V_{Rv}^2 + (V_{Lv} - V_{Cv})^2$$

because $E_0 = \sqrt{2} E_v$, $V_{R0} = \sqrt{2} V_{Rv}$, $V_{L0} = \sqrt{2} V_{Lv}$

and $V_{C0} = \sqrt{2} V_{Cv}$

Similarly, Eq. (xi) can also be written in terms of virtual voltages:

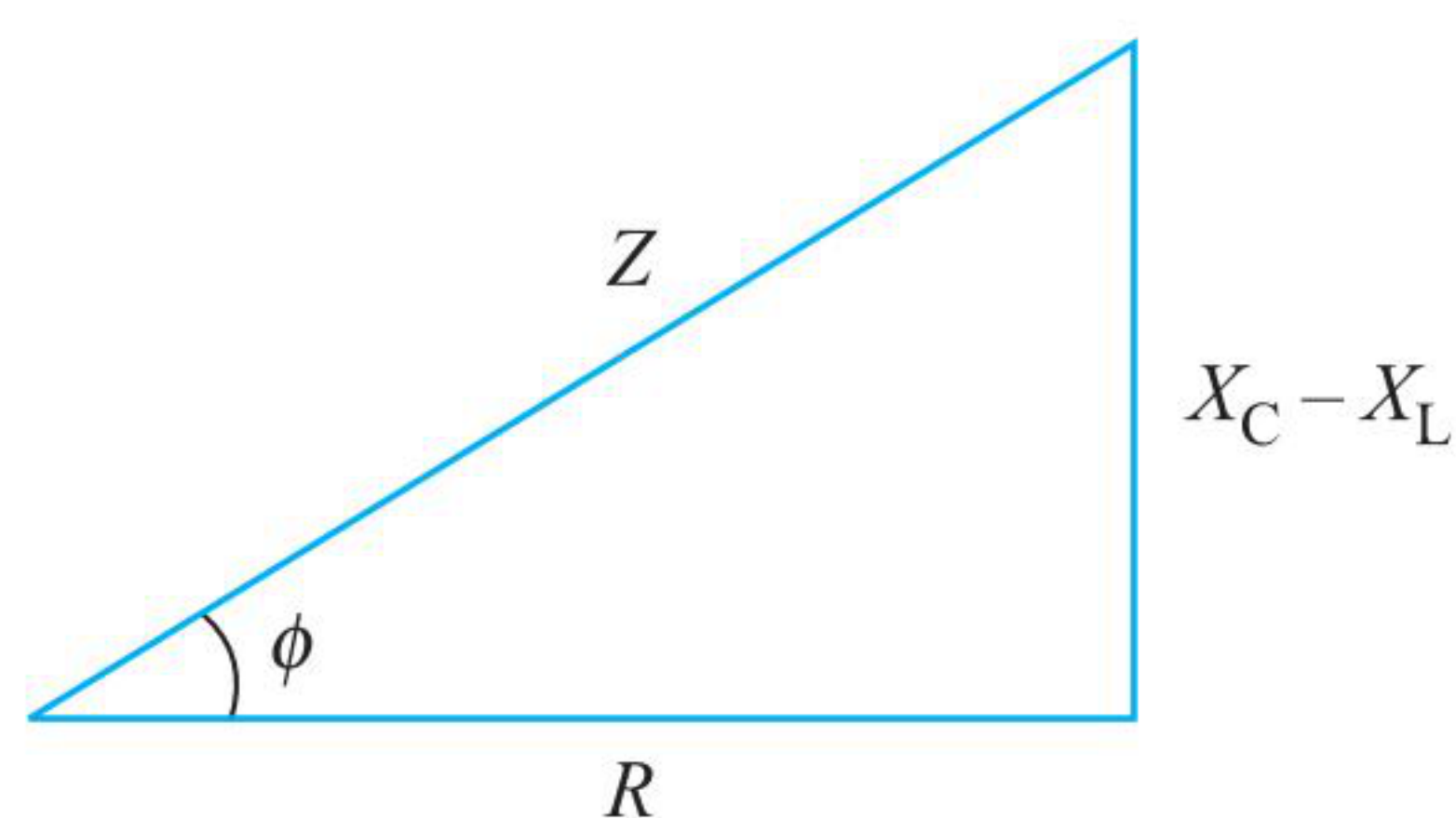
$$\tan \phi = \frac{V_{Cv} - V_{Lv}}{V_{Rv}}$$

We can also write $V_{RV} = I_{VR}$

$$V_{LV} = I_V X_L$$

$$V_{CV} = I_V X_C$$

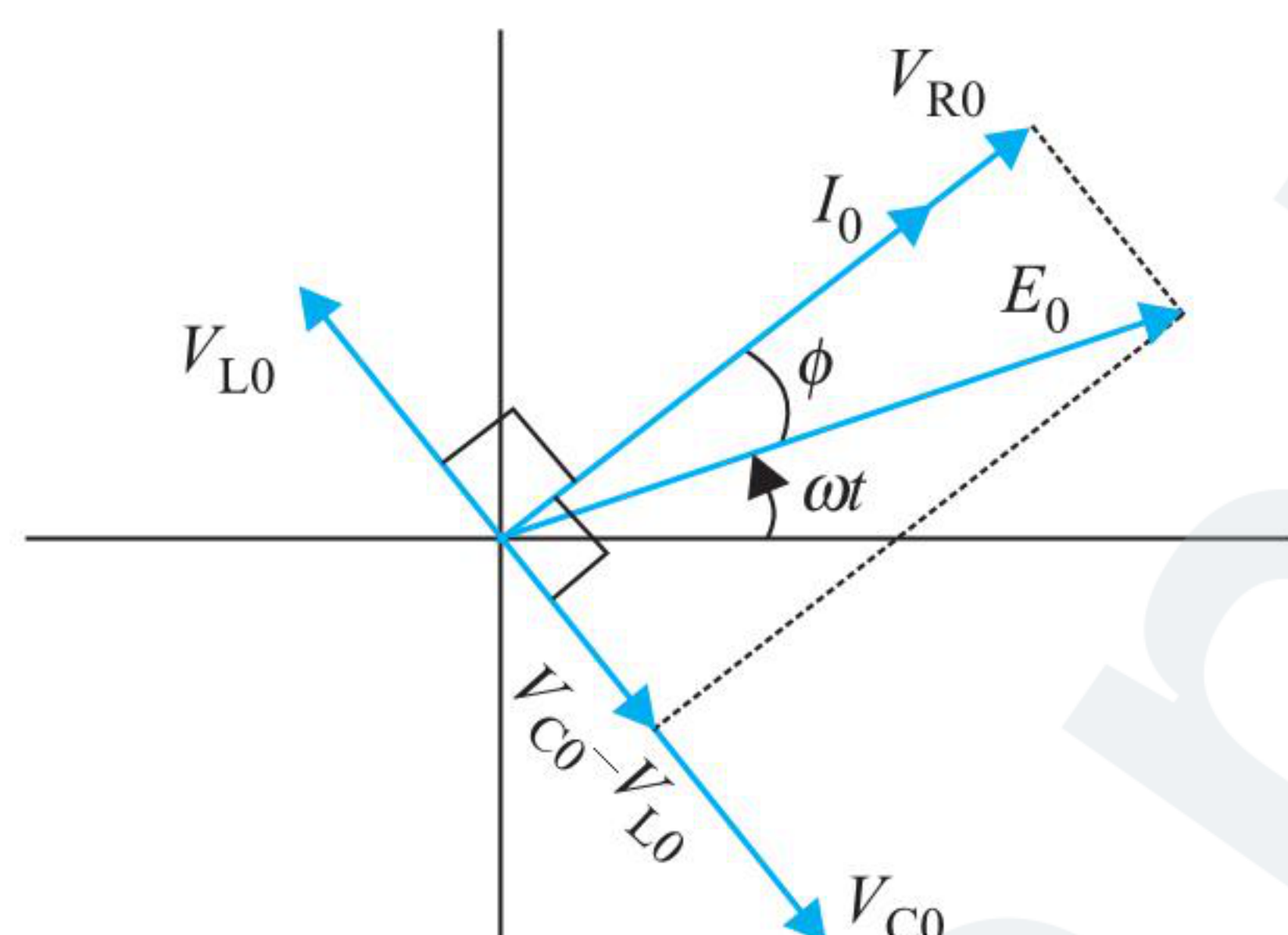
Impedance triangle: Equations (x) and (xii) are shown graphically in figure. It is known as impedance diagram or impedance triangle.



Hypotenuse of this triangle is the impedance of the circuit:

$$\cos \phi = \frac{R}{Z}, \sin \phi = \frac{X_C - X_L}{Z}$$

Figure below shows the phasor diagram of series LCR circuit. Equations (ix) and (xi) can be obtained from this diagram

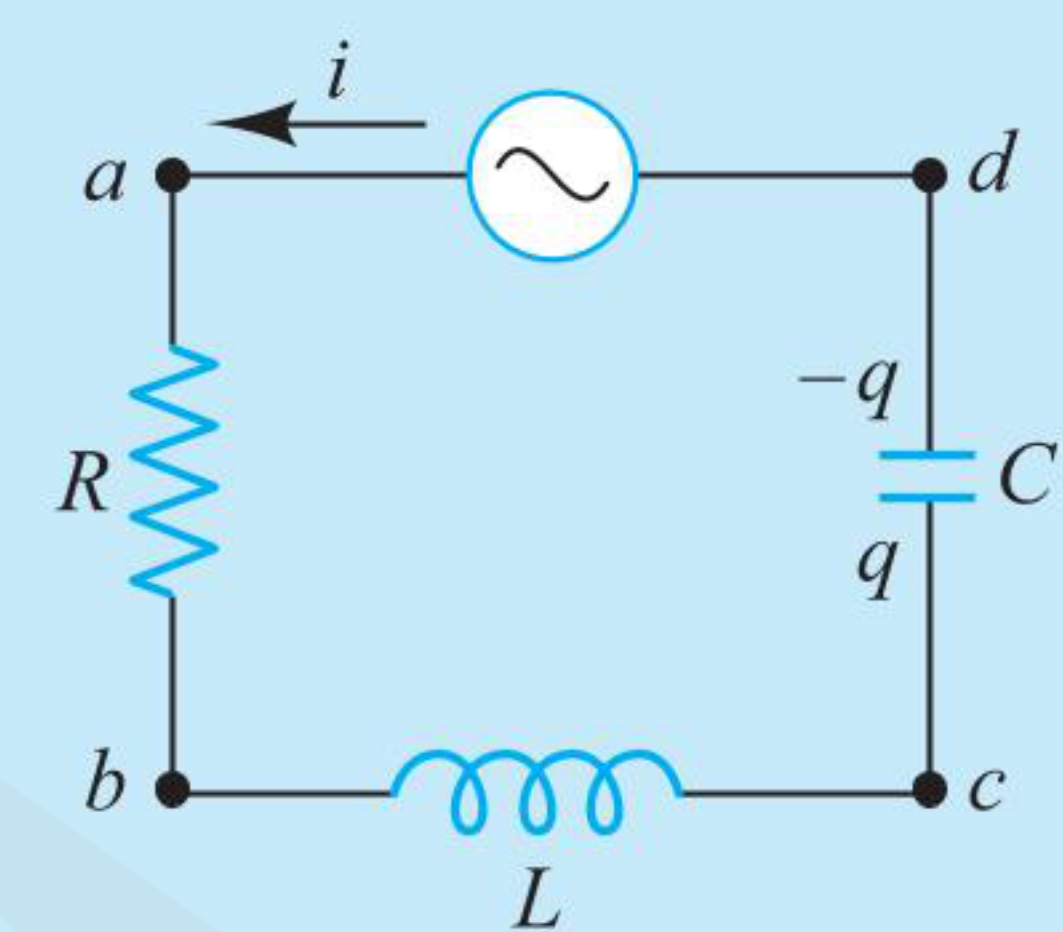


All of the expressions that we have developed for an L-R-C series circuit are still valid if one of the circuit elements is missing. If the resistor is missing, we set $R = 0$; if the inductor is missing, we set $L = 0$, but if the capacitor is missing, we set $C = \infty$, corresponding to the absence of any potential difference ($V_C = q/C = 0$) or any capacitive reactance ($X_C = 1/\omega C = 0$).

Finally, we remark that in this section we have been describing the steady state condition of a circuit, the state that exists after the circuit has been connected to the source for a long time. When the source is first connected, there may be additional voltages and currents, called transients, whose nature depends on the time in the cycle when the circuit is initially completed. A detailed analysis of transients is beyond our scope. They always die out after a sufficiently long time and they do not affect the steady state behaviour of the circuit. But they can cause dangerous and damaging surges in power lines, which is why delicate electronic systems such as computers are often provided with power-line surge protectors.

ILLUSTRATION 6.7

In the series circuit of figure, suppose $R = 300 \Omega$, $L = 60 \text{ mH}$, $C = 0.50 \mu\text{F}$, source amplitude is $E_0 = 50 \text{ V}$ and $\omega = 10,000 \text{ rad s}^{-1}$. Find the reactances X_L and X_C , the impedance Z , the current amplitude I_0 , the phase angle ϕ , and the voltage amplitude across each circuit element.



Sol. The inductive and capacitive reactances are

$$X_L = \omega L = (10,000 \text{ rad/s})(60 \text{ mH}) = 600 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{(10,000 \text{ rad s}^{-1})(0.50 \times 10^{-6} \text{ F})} = 200 \Omega$$

The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(300 \Omega)^2 + (600 \Omega - 200 \Omega)^2} = 500 \Omega$$

With source voltage amplitude $E_0 = 50 \text{ V}$, the current amplitude

$$\text{is } I_0 = \frac{E_0}{Z} = \frac{50 \text{ V}}{500 \Omega} = 0.10 \text{ A}$$

The phase angle ϕ is

$$\phi = \tan^{-1} \frac{X_C - X_L}{R} = \tan^{-1} \left(-\frac{400 \Omega}{300 \Omega} \right) = -53^\circ$$

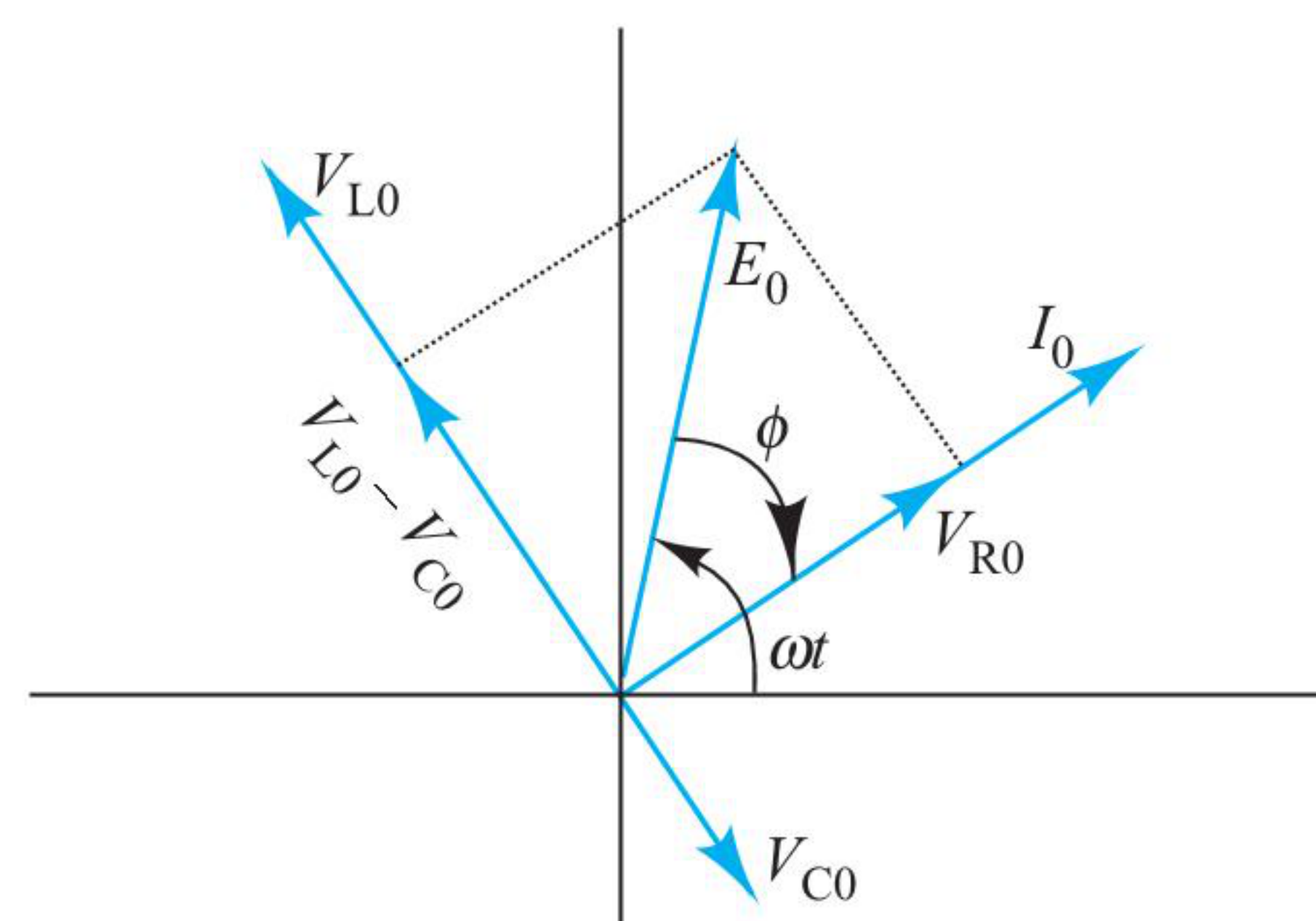
The voltage amplitudes V_{R0} , V_{L0} , and V_{C0} across the resistor, inductor, and capacitor, respectively, are

$$V_{R0} = I_0 R = (0.10 \text{ A})(300 \Omega) = 30 \text{ V}$$

$$V_{L0} = I_0 X_L = (0.10 \text{ A})(600 \Omega) = 60 \text{ V}$$

$$V_{C0} = I_0 X_C = (0.10 \text{ A})(200 \Omega) = 20 \text{ V}$$

Note that $X_L > X_C$ and hence the voltage amplitude across the inductor is greater than that across the capacitor and ϕ is negative. The value $\phi = -53^\circ$ means that the voltage leads the current by 53° ; this is like the situation shown in figure.



Note that the source voltage amplitude $V = 50 \text{ V}$ is not equal to the sum of the voltage amplitude across the separate circuit elements (that is, $50 \text{ V} \neq 30 \text{ V} + 60 \text{ V} + 0 \text{ V}$).

ILLUSTRATION 6.8

For the L-R-C series circuit described in the above Illustration 6.7, describe the time dependence of the instantaneous current and each instantaneous voltage.

Sol. In Illustration 6.7, we found the amplitude of the current and voltages. Now we have to find the expressions for the instantaneous values of the current and voltages. As we learned, the voltage across a resistor is in phase with the current but the voltages across an inductor or capacitor are not. We also learned in this section that ϕ is the phase angle between the source voltage and the current.

The current and all the voltages oscillate with the same angular frequency.

Let us choose

$$E = E_0 \sin \omega t \Rightarrow E = 50 \sin(10,000t)$$

$$\text{then } I = I_0 \sin(10,000t + \phi),$$

$$\text{where } \phi = -53^\circ = -\frac{53 \times \pi}{180} \text{ rad} = -0.93 \text{ rad}$$

$$\Rightarrow I = 0.10 \sin(10,000t - 0.93)$$

Current and voltage are in phase in resistor so

$$V_R = V_{R0} \sin(10,000t - 0.93) = 30 \sin(10,000t - 0.93)$$

In inductor voltage leads current by $\pi/2$ so

$$V_L = V_{L0} \sin[10,000t - 0.93 + (\pi/2)] = 60 \cos(10,000t - 0.93)$$

In capacitor, voltage lags current by $\pi/2$ so

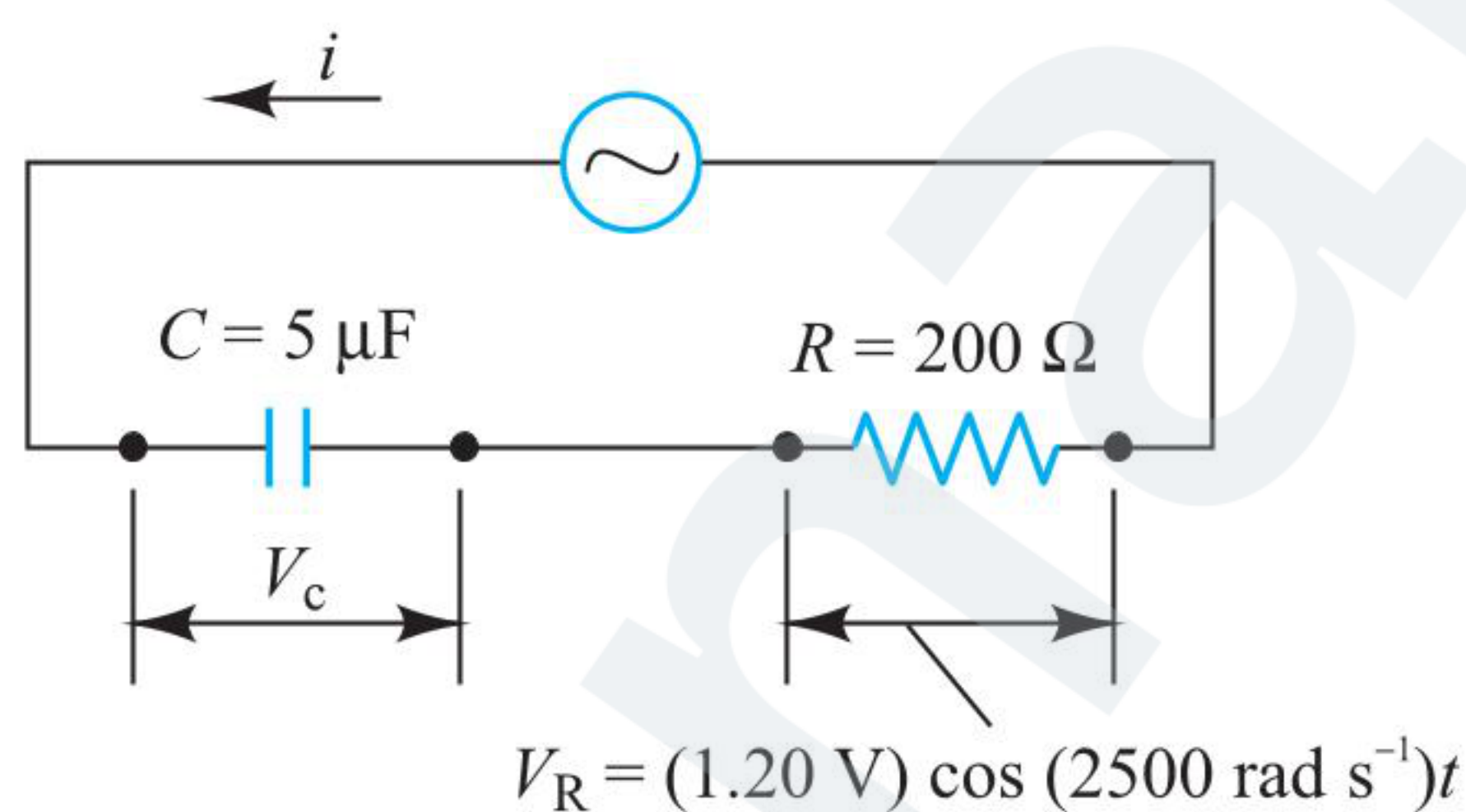
$$\begin{aligned} V_C &= V_{C0} \sin[10,000t - 0.93 - (\pi/2)] \\ &= -20 \cos(10,000t - 0.93) \end{aligned}$$

ILLUSTRATION 6.9

A $200 \, \Omega$ resistor is connected in series with a $5 \, \mu\text{F}$ capacitor. The voltage across the resistor is $V_R = (1.20 \, \text{V}) \cos(2500 \, \text{rad s}^{-1})t$.

- Derive an expression for the circuit current.
- Determine the capacitive reactance of the capacitor.
- Derive an expression for the voltage across the capacitor.

Sol. This is a series circuit. The current is the same through the capacitor as through the resistor. Our target variables are current i , capacitive reactance X_C , and capacitor voltage V_C .



- Using $V_R = iR$, we find that current i in the resistor and through the circuit as a whole is

$$\begin{aligned} i &= \frac{V_R}{R} = \frac{(1.20 \, \text{V}) \cos(2500 \, \text{rad s}^{-1})t}{200 \, \Omega} \\ &= (6.0 \times 10^{-3} \, \text{A}) \cos(2500 \, \text{rad s}^{-1})t \end{aligned}$$

- The capacitive reactance at $\omega = 2500 \, \text{rad s}^{-1}$ is

$$X_C = \frac{1}{\omega C} = \frac{1}{(2500 \, \text{rad s}^{-1})(5.0 \times 10^{-6} \, \text{F})} = 80 \, \Omega$$

- The amplitude V_C of the voltage across the capacitor is

$$V_{C0} = I_0 X_C = (6.0 \times 10^{-3} \, \text{A})(80 \, \Omega) = 0.48 \, \text{V}$$

The instantaneous capacitor voltage V_C is given by

$$\begin{aligned} V_C &= V_{C0} \cos(\omega t - 90^\circ) \\ &= (0.48 \, \text{V}) \cos[(2500 \, \text{rad s}^{-1})t - \pi/2 \text{ rad}] \end{aligned}$$

ILLUSTRATION 6.10

A $200 \, \Omega$ resistor and $1 \, \text{H}$ inductor are joined in series with an ac source of emf $10\sqrt{2} \sin(200t) \, \text{V}$. Calculate the phase difference between emf and current.

Sol. $L = 1 \, \text{H}$, $R = 200 \, \Omega$, $E = 10\sqrt{2} \sin(200t) \, \text{V}$

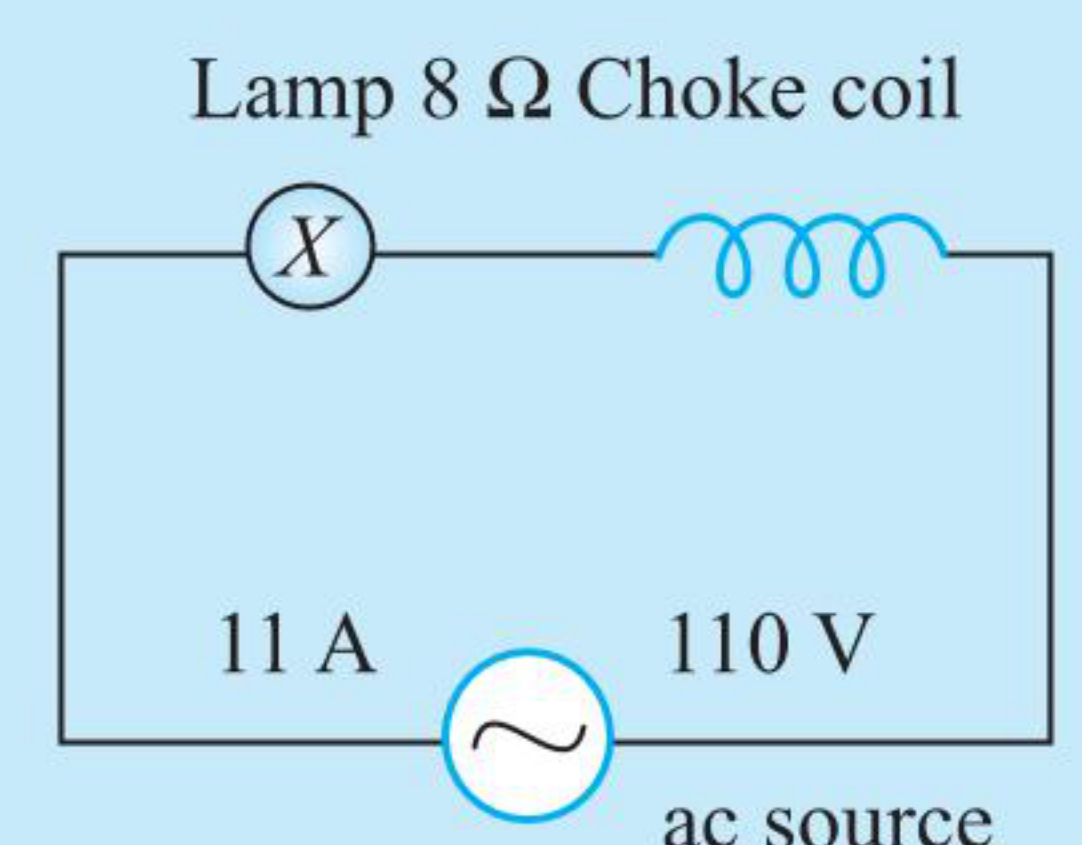
$$\tan \phi = \frac{X_C - X_L}{R} = \frac{-\omega L}{R} = -\frac{200 \times 1}{200} = -1$$

$$\Rightarrow \phi = \tan^{-1}(-1) = -\frac{\pi}{4}$$

ϕ is negative, hence current lags behind voltage by $\pi/4$.

ILLUSTRATION 6.11

A lamp with a resistance of $8 \, \Omega$ is connected to a choke coil as shown in figure. This arrangement is connected to an alternating source of $110 \, \text{V}$. The current in the circuit is $11 \, \text{A}$. The frequency of the ac is $60 \, \text{Hz}$. Find



- the impedance of the circuit, and
- the value of inductive reactance of the choke coil.

Sol. $R = 8 \, \Omega$, $\omega = 2\pi \times 60 = 120\pi \, \text{Hz}$, $I_{\text{rms}} = 11 \, \text{A}$, $E_{\text{rms}} = 110 \, \text{V}$

$$\text{(a) Impedance } (Z) = \frac{E_{\text{rms}}}{I_{\text{rms}}} = \frac{110}{11} = 10 \, \Omega$$

$$\text{(b) } Z^2 = R^2 + X_L^2$$

$$10^2 = 8^2 + X_L^2 \quad \text{or} \quad X_L = \sqrt{(100 - 64)} = \sqrt{36} = 6 \, \Omega$$

ILLUSTRATION 6.12

A resistor of $200 \, \Omega$ and a capacitor of $15.0 \, \mu\text{F}$ are connected in series to a $220 \, \text{V}$, $50 \, \text{Hz}$ source.

- Calculate the current in the circuit.
- Calculate the voltage (rms) across the resistor and the inductor. Is the algebraic sum of these voltages more than the source voltage? If yes, resolve the paradox.

Sol. Given: $R = 200 \, \Omega$, $C = 15.0 \, \mu\text{F} = 15.0 \times 10^{-6} \, \text{F}$

$$E_{\text{rms}} = 220 \, \text{V}, f = 50 \, \text{Hz}$$

- In order to calculate the current, we need the impedance of the circuit.

$$\begin{aligned} Z &= \sqrt{R^2 + X_C^2} = \sqrt{R^2 + (2\pi f C)^{-2}} \\ &= \sqrt{(200 \, \Omega)^2 + (2 \times 3.14 \times 50 \times 15 \times 10^{-6} \, \text{F})^{-2}} \\ &= \sqrt{(200 \, \Omega)^2 + (212 \, \Omega)^2} = 291.5 \, \Omega \end{aligned}$$

Therefore, the current in the circuit is

$$I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{220 \text{ V}}{291.5 \Omega} = 0.755 \text{ A}$$

(b) Since the current is the same throughout the circuit,

$$V_R = I_{\text{rms}} R = (0.755 \text{ A}) (200 \Omega) = 151 \text{ V}$$

$$V_C = I_{\text{rms}} X_C = (0.755 \text{ A}) (212 \Omega) = 160 \text{ V}$$

The algebraic sum of the two voltages V_R and V_C is 311 V, which is more than the source voltage of 220 V. How to resolve this paradox? You may recall that the two voltages are not in the same phase. Therefore, they cannot be added like the ordinary numbers. The two voltages are out of phase by 90° . Therefore, the total of these voltages must be obtained using the Pythagoras theorem:

$$V_{RC} = \sqrt{V_R^2 + V_C^2} = \sqrt{(151 \text{ V})^2 + (160 \text{ V})^2} = 220 \text{ V}$$

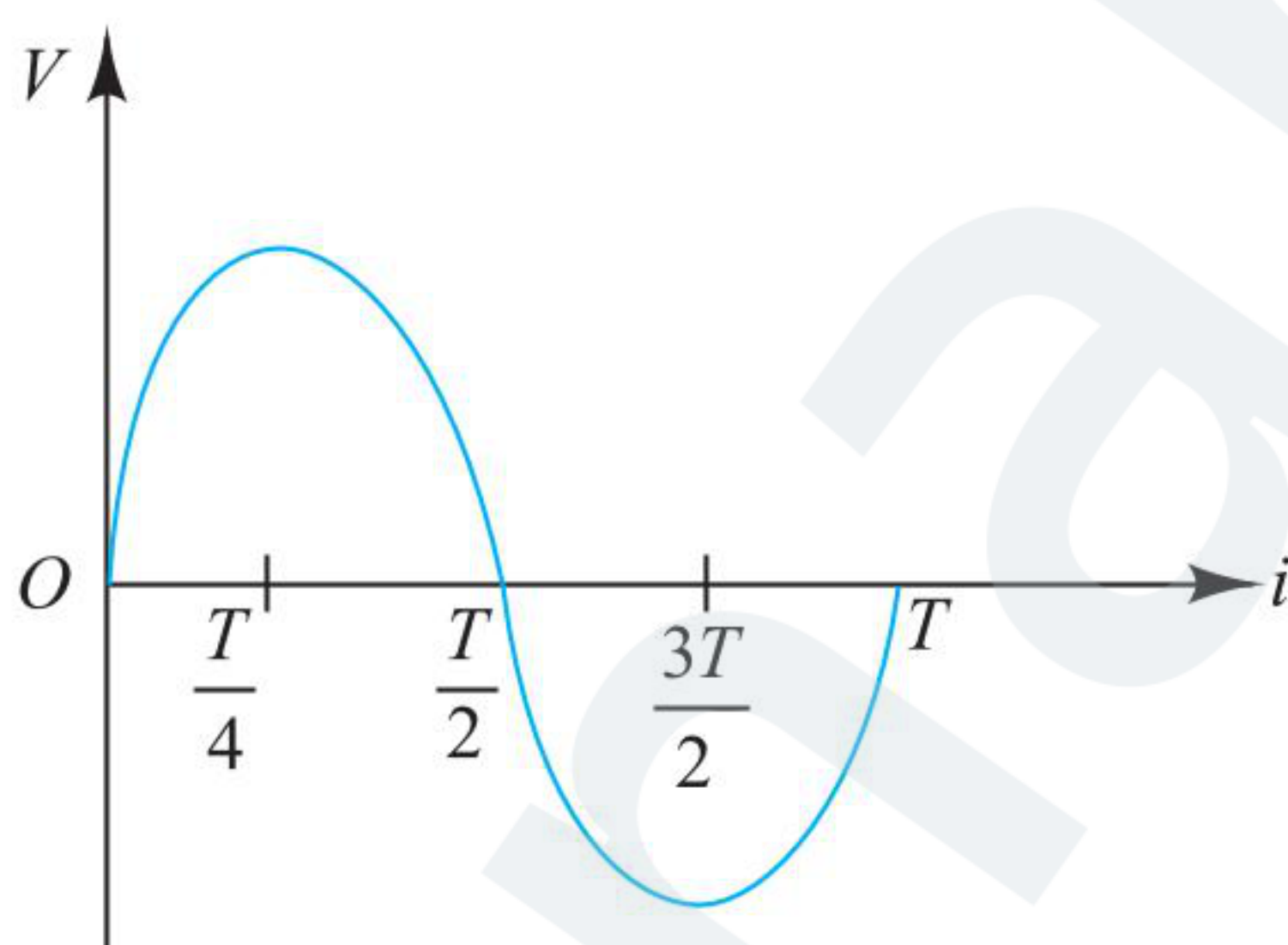
Thus, if the phase difference between two voltages is properly taken into account, the total voltage across the resistor and the inductor is equal to the voltage of the source.

ILLUSTRATION 6.13

In an LR series circuit, a sinusoidal $V = V_0 \sin \omega t$ is applied. It is given that $L = 35 \text{ mH}$, $R = 11 \Omega$, $V_{\text{rms}} = 220 \text{ V}$, $\omega/2\pi = 50 \text{ Hz}$ and $\pi = 22/7$. Find the amplitude of the current in steady state and obtain the phase difference between the current and the voltage. Also plot the variation of the current for one cycle on the given graph.

Sol. Inductive reactance,

$$\begin{aligned} X_L &= \omega L = 2\pi \times 50 \times 35 \times 10^{-3} \\ &= 2 \times \frac{22}{7} \times 50 \times 35 \times 10^{-3} = 11 \Omega \end{aligned}$$



Therefore, impedance of circuit

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{11^2 + 11^2} = 11\sqrt{2} \Omega$$

Given $V_{\text{rms}} = 220 \text{ V}$

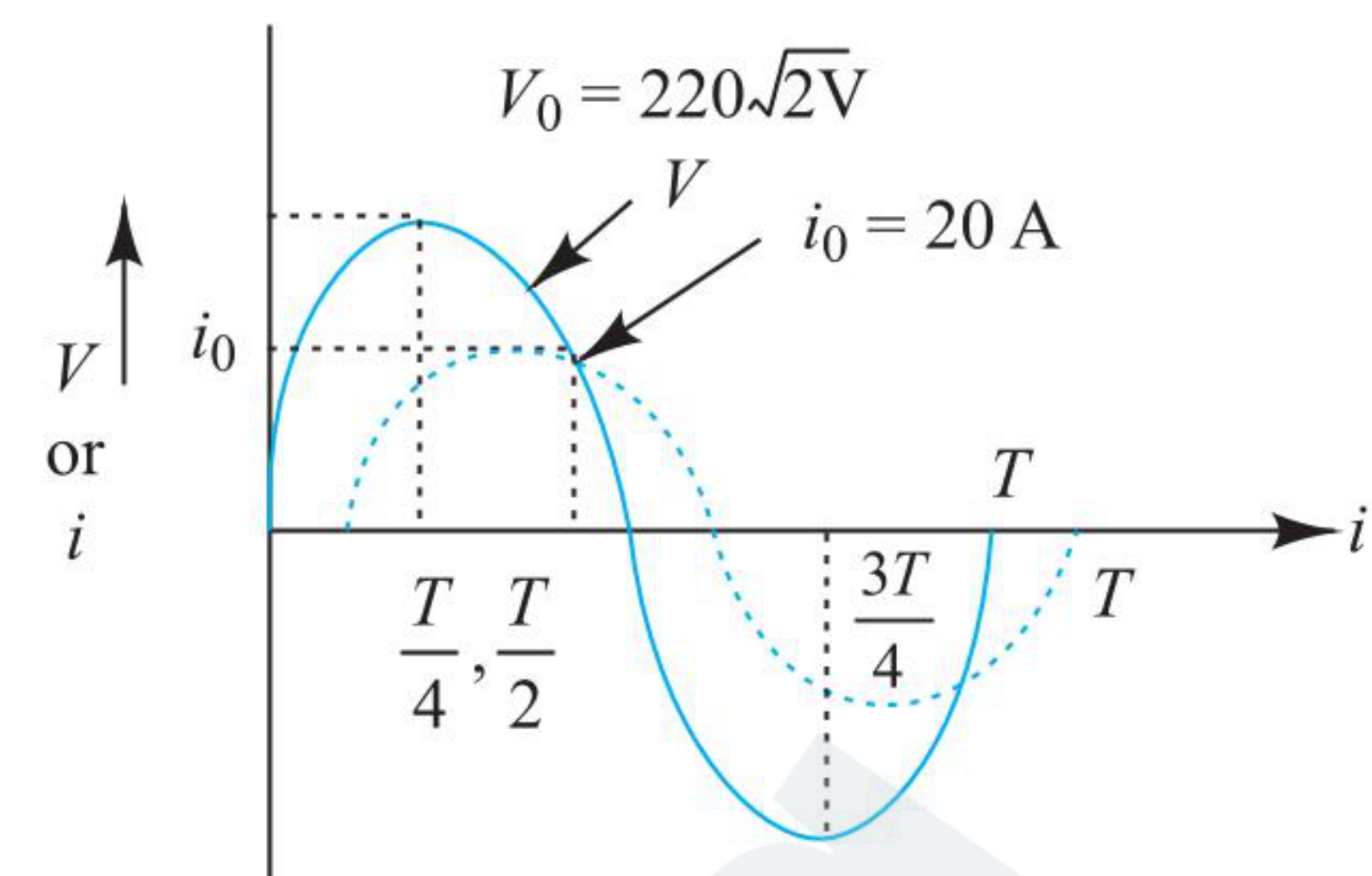
Amplitude of voltage $V_0 = \sqrt{2} V_{\text{rms}} = 220 \sqrt{2} \text{ V}$

Amplitude of current $i_0 = \frac{V_0}{Z} = \frac{220\sqrt{2}}{11\sqrt{2}} = 20 \text{ A}$

Phase lag of current over voltage

$$\phi = \tan^{-1} \frac{\omega L}{R} = \tan^{-1} \frac{11}{11} = \tan^{-1}(1) = \frac{\pi}{4}$$

Voltage and current variations are shown in figure.



The voltage and the current are expressed as $V = 220 \sqrt{2} \sin \omega t$

$$I = 20 \sin \left(\omega t - \frac{\pi}{4} \right)$$

POWER IN SERIES LCR CIRCUITS

Alternating currents play a central role in the system for distributing, converting, and using electrical energy, so it is important to look at power relationships in ac circuits.

The power in an electric circuit is the rate at which electrical energy is consumed in the circuit. Let an emf $E = E_0 \sin \omega t$ be applied to a series LCR circuit. The current in the circuit is $I = I_0 \sin(\omega t + \phi)$, where ϕ is the phase difference between current and voltage. The instantaneous power is given by

$$P = EI = E_0 I_0 \sin \omega t \sin(\omega t + \phi)$$

If the average power consumed in the cycle from time t_1 to t_2 is P_{av} , then

$$P_{\text{av}}(t_2 - t_1) = \int_{t_1}^{t_2} P dt$$

Average power over 1 cycle, i.e., $t_1 = 0$ to $t_2 = T$ is given by

$$\begin{aligned} P_{\text{av}} T &= \int_0^T E_0 I_0 \sin \omega t \sin(\omega t + \phi) dt \\ \Rightarrow P_{\text{av}} T &= E_0 I_0 \int_0^T \left(\sin^2 \omega t \cos \phi + \frac{\sin 2\omega t}{2} \sin \phi \right) dt \\ &\left[\begin{array}{l} \text{remember } \int_0^T \sin^2 \omega t dt = \frac{T}{2} \\ \text{and } \int_0^T \sin 2\omega t dt = 0 \end{array} \right] \end{aligned}$$

$$\Rightarrow P_{\text{av}} T = \frac{E_0 I_0}{2} \cos \phi T$$

$$\begin{aligned} \Rightarrow P_{\text{av}} &= E_v I_v \cos \phi \\ &= P_{\text{ap}} \cos \phi, \end{aligned}$$

where $P_{\text{ap}} = E_v I_v \rightarrow$ apparent power or virtual power

and $\cos \phi = \frac{P_{\text{av}}}{P_{\text{ap}}} \rightarrow$ power factor

$$\text{also } P_{\text{av}} = E_v I_v \cos \phi = (I_v Z) I_v \frac{R}{Z} = I_v^2 R$$

For pure resistive circuit: $\phi = 0$ and $Z = R$

$$P_{\text{av}} = E_v I_v = (I_v Z) I_v = I_v^2 Z = I_v^2 R$$

For pure inductive circuit: $\phi = -\frac{\pi}{2}$

$$\text{So } P_{\text{av}} = 0$$

Therefore, the average power over a complete cycle of ac through an ideal inductor is zero. Actually, whatever energy is needed in building up current in inductance is returned back during the decay of current.

$$\text{For pure capacitive circuit: } \phi = \frac{\pi}{2}$$

$$P_{\text{av}} = 0$$

In this case too, the average power is zero. Actually, whatever energy is needed in building up the voltage across capacitor is returned to the source during discharging of capacitor.

So average power consumed in pure inductive or pure capacitive circuit is zero. So there is no loss of energy in the inductor or capacitor. But in a resistance, loss of energy occurs. It cannot store the energy like inductor or capacitor.

The current through pure L or C , which dissipates no power is called idle current or wattless current. L and C are most suitable for controlling the current in ac circuits.

WATTLSS COMPONENT OF CURRENT

$I_v \sin \phi$ is known as wattless component of current.

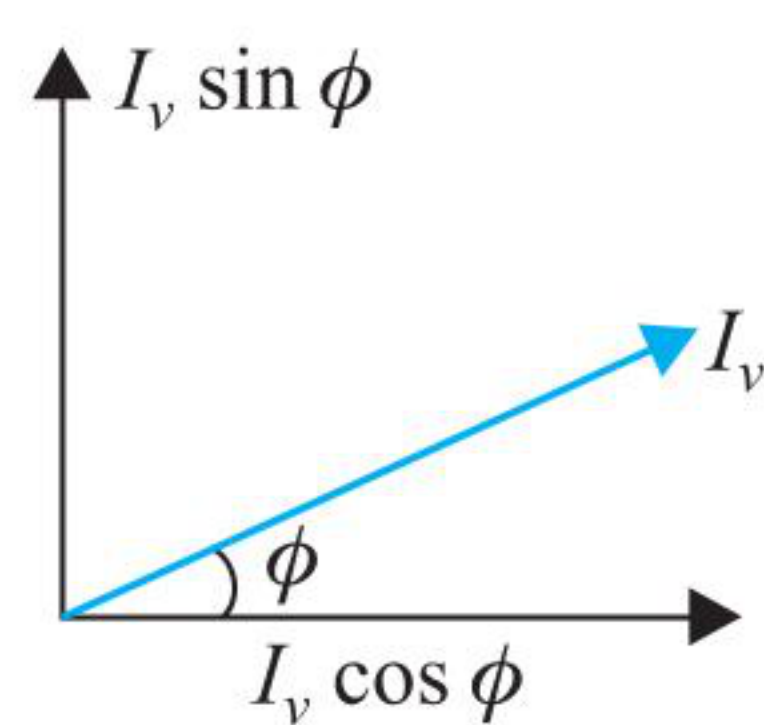


ILLUSTRATION 6.14

The potential difference E and current I flowing through the ac circuit is given by $E = 5 \cos(\omega t - \pi/6)$ V and $I = 10 \sin \omega t$ A. Find the average power dissipated in the circuit.

Sol.

$$E = 5 \cos[\omega t - (\pi/6)], I = 10 \sin \omega t = 10 \cos[\omega t - (\pi/2)]$$

Phase difference between E and I is given by:

$$\phi = \frac{\pi}{2} - \frac{\pi}{6} = \frac{2\pi}{6} = \frac{\pi}{3}$$

Average power dissipated:

$$P = \frac{E_0 I_0}{2} \cos \phi = \frac{5 \times 10}{2} \times \frac{1}{2} = 12.5 \text{ W}$$

CHOKE COIL

Choke coil is required to reduce the current in a given circuit with a minimum wastage of energy when the current is derived at constant voltage from a given supply. In dc circuits, a resistance is used for this purpose but there is a wastage of energy $i^2 R$. Similarly, if in ac circuit, resistance is used, the wastage of energy will also be $i^2 R$. However, with an alternating supply, a more economical method to reduce the current without any wastage is by introducing an inductance. The inductance sets up a back emf which opposes the applied emf. So, the current is reduced. When the resistance of the inductance is negligible, no power is

dissipated. Such an inductance coil used in ac circuits to limit the current without dissipation of power is called a choke coil.

RESONANCE IN SERIES LCR CIRCUIT

When the current in the series LCR circuit is maximum possible, then the circuit is said to be in resonance. We know that in a series LCR circuit, amplitude of current is given by

$$I_0 = \frac{E_0}{Z} = \frac{E_0}{\sqrt{R^2 + (X_L - X_C)^2}}$$

where $X_L = \omega L$ and $X_C = 1/\omega C$

If we vary the angular frequency ω of the source, X_L and X_C both vary and I_0 also varies. I_0 is maximum when Z is minimum. Z is minimum when $X_L = X_C$. Let at $\omega = \omega_0$ when $X_L = X_C$

$$\Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\Rightarrow 2\pi f_0 = \frac{1}{\sqrt{LC}} \Rightarrow f_0 = \frac{1}{2\pi\sqrt{LC}}$$

So at $\omega = \omega_0$, maximum possible current flows in circuit and the circuit is said to be in resonance. ω_0 is known as resonance frequency. At resonance frequency, impedance is minimum which is given by $Z_{\text{min}} = R$ and at resonance, the value of I_0 is maximum which is given by

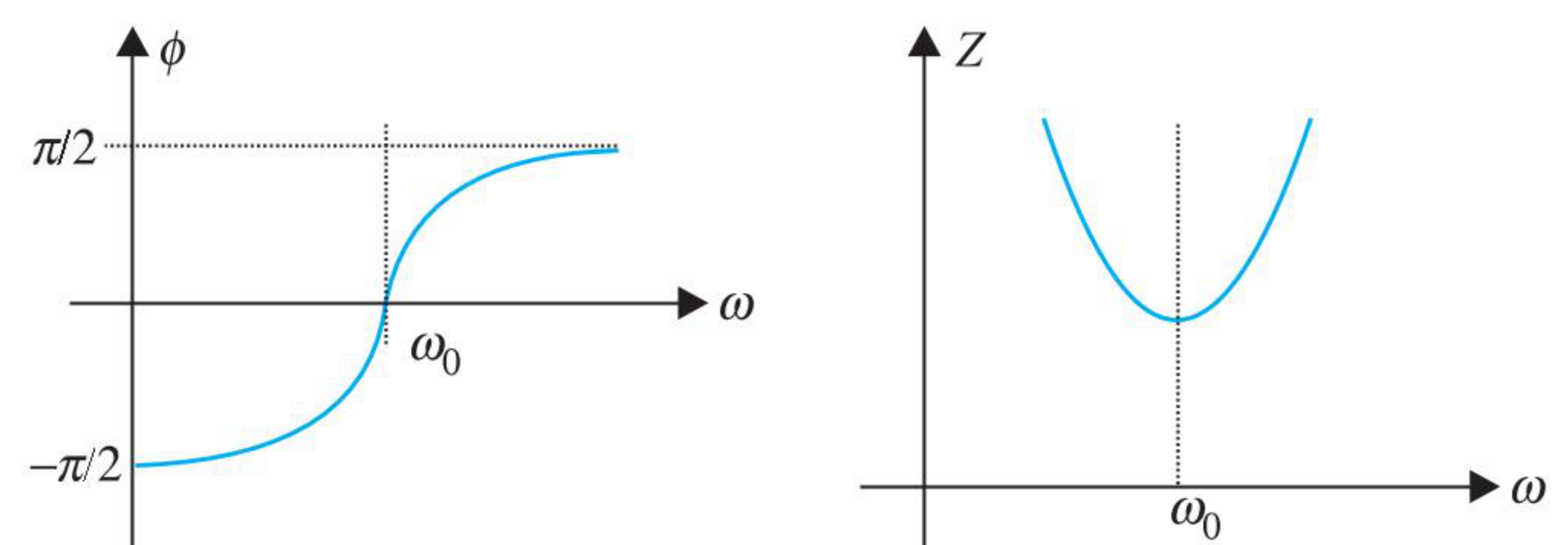
$$I_{0(\text{max})} = \frac{E_0}{R}$$

The variation of I_0 with ω is as shown in figure.

A series resonance circuit is used in tuning circuits. The tuning circuits of radio and TV sets, etc. are series LCR circuits.

The instantaneous voltages across L and C always differ in phase by 180° ; they have opposite sign at each instant. At the resonance frequency, $X_L = X_C$ and the voltages $V_L = V_C$. Then the instantaneous voltages across L and C add to zero at each instant, and the total voltage across the $L-C$ combination is exactly zero. The voltage across the resistor is then equal to the source voltage. So at the resonance frequency, the circuit behaves as if the inductor and the capacitor were not there at all.

This variation of ϕ and Z with angular frequency is shown in figure.



Note:

At resonance power factor is maximum. It is given by

$$\cos \phi = \frac{Z}{R} = \frac{R}{R} = 1 \quad (\because Z = R \text{ at resonance})$$

Thus, power factor is unity at resonance. Maximum power is dissipated during resonance.

Q-FACTOR (OR QUALITY FACTOR) OF RESONANCE

The quality factor Q is a parameter which is used to describe the sharpness of resonance curve. The quality factor Q of a series resonance circuit is defined as the ratio of voltage drop across inductor (or capacitor) at resonance to the applied voltage. Thus,

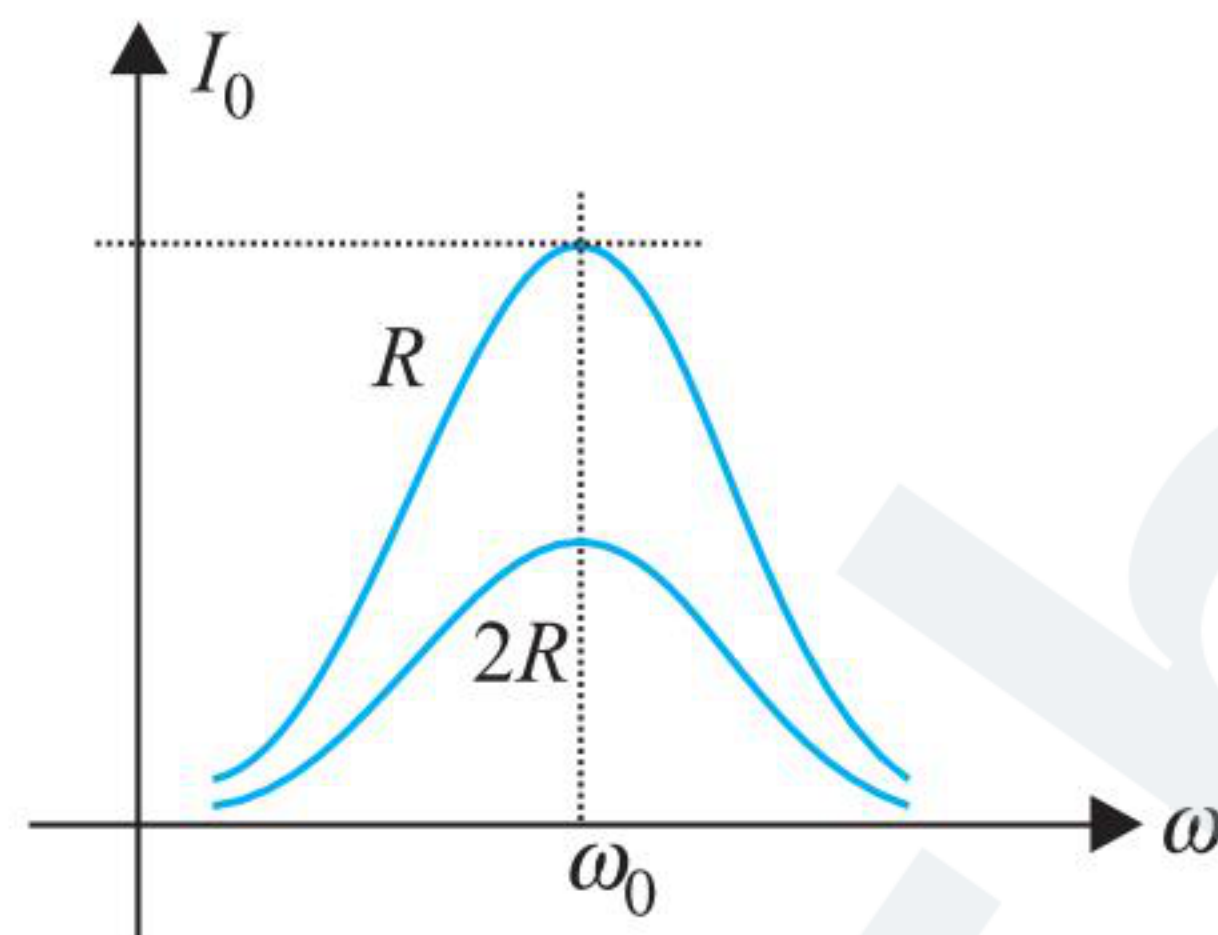
$$Q = \frac{\text{voltage across } L \text{ (or } C \text{) at resonance}}{\text{applied voltage}} \quad (i)$$

$$\begin{aligned} &= \frac{I_v \omega_0 L}{I_v R} = \omega_0 \left(\frac{L}{R} \right) \\ &= \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned} \quad (ii)$$

The rapidity with which the current falls from its resonant value $I_{0(\max)} = E_0/R$ with change in applied frequency is known as sharpness of resonance. More quickly the current falls from its maximum value with change in frequency, more is the sharpness of resonance.

If Q -factor is high, the current will respond to very narrow range of frequencies and the resonance will be sharper. Usual value of Q is 10 to 100 but it can go upto 200. Q -factor is a dimensionless quantity. Equation (ii) shows that when R increases, Q -factor of the circuit decreases.

In figure, two curves are plotted, one when the circuit resistance is low, i.e., R , and the other when the circuit resistance is high, i.e., $2R$. We observe the following points:



- The peak of the curve depends on the resistance of the circuit. When R is low, the peak is high and vice-versa. The peak is related to sharpness of resonance. More is height of peak, more sharp is the resonance.
- It is clear from figure that the width of resonance curve increases on increasing the value of R . Therefore, the resonance curve becomes flatter.
- The series resonant circuit is sometimes called the acceptor circuit. The reason is that impedance of the circuit is minimum at resonance and, due to this fact, it readily accepts that current out of the many currents whose frequency is equal to its resonant frequency.

Alternate method: The quality factor Q is also defined as 2π times the ratio of the energy stored in L (or C) to the average energy loss per period. Therefore,

$$Q = 2\pi \left[\frac{\text{maximum energy stored in circuit}}{\text{energy loss per period}} \right]$$

The maximum energy stored in inductor: $U = \frac{1}{2} L (I_0)^2$

The energy dissipated per second: $P_R = I_{\text{rms}}^2 R = \frac{I_0^2 R}{2}$

Energy dissipated per time period: $U_R = \frac{I_0^2 R}{2} \times T$

Substituting these values, we get

$$\begin{aligned} Q &= 2\pi \left[\frac{\frac{1}{2} L I_0^2}{\left(\frac{I_0^2 R}{2} \right) T} \right] = \frac{2\pi}{T} \times \left(\frac{L}{R} \right) \\ &= \omega_0 \left(\frac{L}{R} \right) = \frac{1}{\sqrt{LC}} \times \frac{L}{R} \quad \left(\because \omega_0 = \frac{1}{\sqrt{LC}} \right) \\ &= \frac{1}{R} \sqrt{\frac{L}{C}} \end{aligned}$$

So, the Q -factor of the circuit varies inversely as R . Thus, at resonance, the voltage drop across inductance or capacitance is Q -times the applied voltage.

ANOTHER DEFINITION OF Q-FACTOR

We know that current in series LCR is given by

$$I_v = \frac{E_v}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2}} \quad \dots(i)$$

Now consider the value of I_v for which power dissipated in the circuit is half of maximum power. For this we have

$$P = \frac{P_{\max}}{2} = I_v^2 R, \quad \text{where} \quad P_{\max} = I_{v(\max)}^2 R$$

$$\text{From above: } I_v = \frac{I_{v(\max)}}{\sqrt{2}} = \frac{E_v}{R\sqrt{2}}$$

Putting the values of I_v in Eq. (i):

$$\begin{aligned} \frac{E_v}{R\sqrt{2}} &= \frac{E_v}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2}} \\ \Rightarrow \sqrt{2} R &= \sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2} \\ \Rightarrow 2R^2 &= R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2 \\ \Rightarrow R^2 &= \left(\frac{1}{\omega C} - \omega L \right)^2 \\ \Rightarrow \frac{1}{\omega C} - \omega L &= \pm R \\ \Rightarrow 1 - \omega^2 LC &= \pm R\omega C \\ \Rightarrow LC\omega^2 \pm RC\omega - 1 &= 0 \end{aligned}$$

From here, we get

$$\omega = \frac{\mp RC \pm \sqrt{R^2 C^2 + 4LC}}{2LC}$$

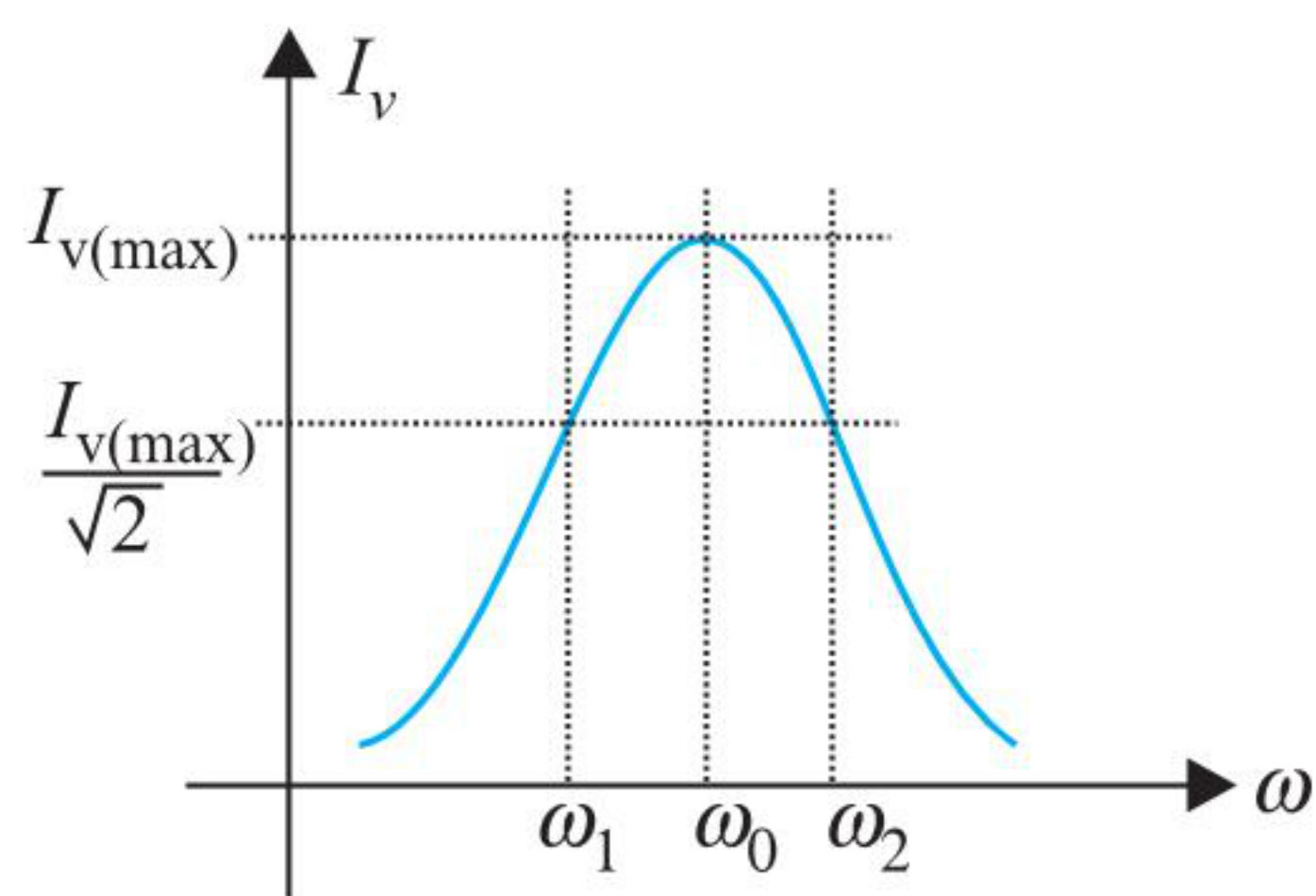
As ω cannot be negative, so

$$\omega = \frac{\mp RC + \sqrt{R^2 C^2 + 4LC}}{2LC}$$

We get the following two values of ω

$$\omega_1 = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC} \text{ and } \omega_2 = \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC}$$

These values are indicated in figure also.



These two frequencies ω_1 and ω_2 are known as half power frequencies. From these, we can get

$$\omega_1\omega_2 = \frac{1}{LC} \text{ and } \omega_2 - \omega_1 = \frac{R}{L}$$

$\omega_2 - \omega_1 = 2\Delta\omega$ is known as band width.

Now the ratio $\frac{\omega_0}{2\Delta\omega}$ is also known as Q -factor. Thus,

$$Q = \frac{\omega_0}{2\Delta\omega} = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}}$$

Lesser the band width, sharper is the resonance.

ILLUSTRATION 6.15

A series LCR circuit with $L = 0.12$ H, $C = 480$ nF, and $R = 23$ Ω is connected to a 230-V variable frequency supply.

- What is the source frequency for which current amplitude is maximum? Find this maximum value.
- What is the source frequency for which average power absorbed by the circuit is maximum? Obtain the value of maximum power.
- For which frequencies of the source is the power transferred to the circuit half the power at resonant frequency?
- What is the Q -factor of the circuit?

Sol.

- Current amplitude is maximum at resonance and source frequency at resonance is given by

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.12 \times 480 \times 10^{-9}}} = 663.14 \text{ Hz}$$

$$\text{and } \omega_0 = 2\pi f_0 = 4166.66 \text{ rad s}^{-1}$$

The maximum current amplitude is given by

$$I_{0(\max)} = \frac{E_0}{R} = \frac{230\sqrt{2}}{23} = 14.14 \text{ A}$$

- Average power absorbed by the circuit is maximum at resonance for which frequency is calculated above as 663.14 Hz. Maximum power absorbed is given by

$$P_{\max} = \frac{E_v^2}{R} = \frac{(230)^2}{23} = 2300 \text{ W}$$

$$(c) \quad P = E_v I_v \cos \phi = E_v I_v \frac{R}{Z} = \frac{E_v^2}{Z^2} R \quad \text{and} \quad P_{\max} = \frac{E_v^2}{R}$$

$$\text{Given } P = P_{\max}/2$$

$$\Rightarrow \frac{E_v^2 R}{Z^2} = \frac{1}{2} \frac{E_v^2}{R} \Rightarrow Z^2 = 2R^2$$

$$\Rightarrow R^2 + (X_C - X_L)^2 = 2R^2$$

$$\Rightarrow X_C - X_L = \pm R$$

Solve to get

$$\omega_1 = \frac{-RC + \sqrt{R^2C^2 + 4LC}}{2LC} = 4071.9 \text{ rad s}^{-1}$$

$$\text{and } \omega_2 = \frac{RC + \sqrt{R^2C^2 + 4LC}}{2LC} = 4263.6 \text{ rad s}^{-1}$$

- Q -factor is given by

$$Q = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{1}{23} \sqrt{\frac{0.12}{480 \times 10^{-9}}} = 21.74$$

CHOKE COIL

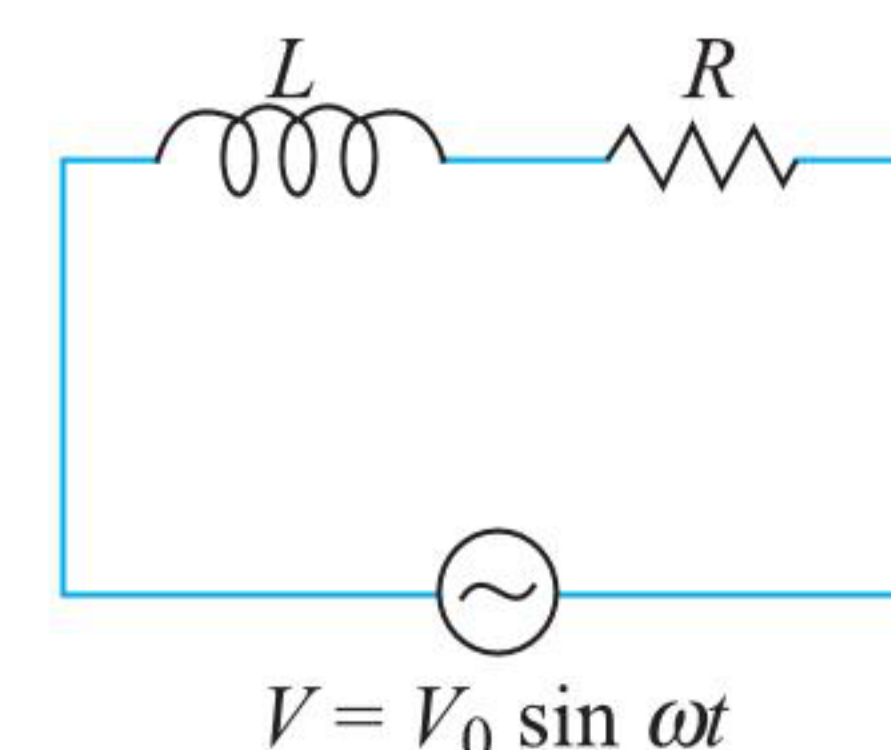
In case of circuits with d.c. source, whenever we need to reduce the value of the current in a circuit, then its value can be reduced by using a rheostat. But in doing so, a power equal to $I^2 R$ will be wasted in the form of heat where I is the current flowing through the circuit whose resistance is R .

We avoid unwanted energy loss in AC circuits, while regulating voltage/current, by using an inductor coil in place of a resistance. Such an inductor coil having small resistance (ideally zero) is known as a choke coil.

A choke coil is simply an inductor with large inductance which is used to reduce current in a.c. circuits without much loss of energy.

Principle: The choke coil is made of thick insulated copper wire wound closely in a large number of turns over a soft-iron laminated core. It offers a large reactance $X_L = 2\pi f L$ to the flow of a.c. and hence current is reduced. Laminated core reduces losses due to eddy currents.

Working: A choke is put in series across an electrical appliance of resistance R and is connected to an a.c. source. This forms an LR-circuit.



Average power dissipated per cycle in the circuit is

$$P_{av} = V_{\text{eff}} I_{\text{eff}} \cos \phi = V_{\text{eff}} I_{\text{eff}} \cdot \frac{R}{\sqrt{R^2 + \omega^2 L^2}}$$

Inductance L of the choke coil is very large so that $R \ll \omega L$. Then

$$\text{Power factor, } \cos \phi = \frac{R}{\omega L} \rightarrow 0$$

i.e., Average power dissipated by the coil is very small. As impedance, $Z = \sqrt{R^2 + \omega^2 L^2}$ is large, so current is reduced without appreciable wastage of power.

$$\text{Current in the circuit is } I = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (\omega L)^2}}$$

The potential difference across R is

$$V_R = IR = V \frac{R}{\sqrt{R^2 + (\omega L)^2}}$$

Thus, we have successfully reduced the potential difference across the resistance by increasing the impedance. But this does not result in any extra energy loss as the inductor does not consume power.

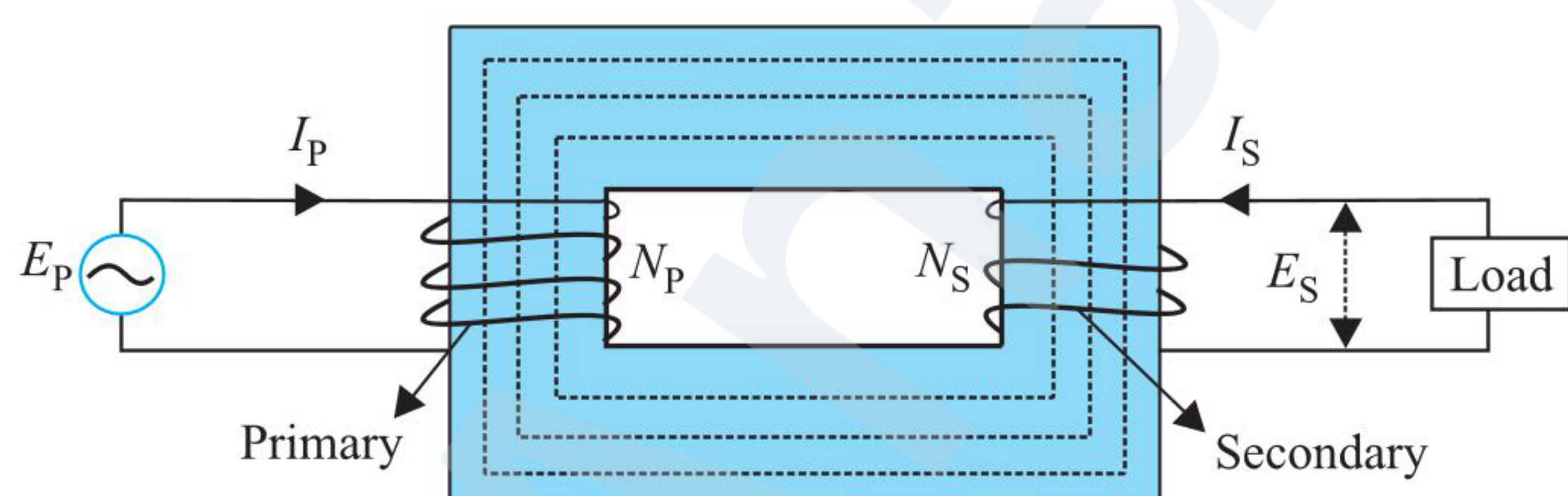
Uses: The most common use of choke coil is in the fluorescent tubes with a.c. mains. Choke coils are also used in various electronic circuits, mercury lamp and in sodium vapour lamp.

Those chokes which are used at low frequencies have an iron core and are called audio frequency (a.f.) chokes. Chokes which are to be used at high frequencies have air as core. Such chokes are called radio frequency (r.f.) chokes.

TRANSFORMERS

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. For a long distance power transmission, it is desirable to use as high a voltage and as small a current as possible; this reduces i^2R losses in the transmission lines, and as smaller wires can be used, it saves material costs. Present day transmission lines routinely operate at rms voltages of the order of 500 kV. On the other hand, safety considerations and insulation requirements dictate relatively low voltages in generating equipment and in household and industrial power distribution. The standard voltage for household wiring is 120 V in the United States and Canada and 240 V in many other countries. The necessary voltage conservation is accomplished by the use of transformers.

A transformer is an ac device which transfers electric power from one circuit to another. It can raise or lower the voltage in a circuit but with a corresponding decrease or increase in current. Here it is worth mentioning that while transferring the power, frequency is not altered. Figure shows a basic transformer.



As evident, any transformer has two coils electrically insulated but magnetically linked. By magnetic link we mean that the two coils are wound on a common core. The energy from one coil is transferred to other coil by means of magnetic coupling. The coil which receives energy from an ac source is called primary and the coil which delivers the energy to the load is called secondary.

A transformer operates on the principle of mutual induction. When an alternating voltage is applied to the primary coil, an alternating voltage is set up in it. As the winding is linked with a magnetic emf, it produces an alternating flux in the core. This alternating flux links with the turns of the secondary coil. Since

the flux is alternating in magnitude, it induces a mutually induced emf in secondary coil of the same frequency as the flux. This follows from Faraday's law of electromagnetic induction, i.e., $e = -M(di/dt)$. Because of this induced emf, the secondary coil is capable of supplying current and, hence, energy.

Theory and working: Let N_P and N_S be the number of turns in the primary and secondary coils, respectively. Here we assume that the entire magnetic flux of the primary coil is linked to the secondary coil. This results in the same magnetic flux per turn linked with each of the two coils. Let ϕ be the flux linked with each turn of either coil at any instant. Then

$$\text{emf induced in primary coil, } E_P = -\frac{N_P d\phi}{dt}$$

$$\text{emf induced in secondary coil, } E_S = -\frac{N_S d\phi}{dt}$$

If an alternating emf E_P is applied across that primary coil, then Kirchhoff's loop law applied to primary circuit gives

$$E_P - N_P \frac{d\phi}{dt} = 0$$

$$\text{or } E_P = N_P \frac{d\phi}{dt} \quad \dots(i)$$

Here, we neglect the resistance of primary circuit.

If E_S represents the alternating emf in secondary coil, then

$$E_S = -N_S \frac{d\phi}{dt} \quad \dots(ii)$$

From Eqs. (i) and (ii), we get

$$\frac{E_S}{E_P} = -\frac{N_S}{N_P} \quad \text{or} \quad E_S = -\frac{N_S}{N_P} E_P$$

The negative sign shows that E_S is 180° out of phase with E_P . Ignoring negative sign,

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} \quad \dots(iii)$$

Let I_P and I_S be the currents at any instant in primary and secondary coils, respectively. Considering the transformer to be ideal (no loss of energy by any means),

$$E_P I_P = E_S I_S \Rightarrow \frac{E_S}{E_P} = \frac{I_P}{I_S} \quad \dots(iv)$$

From Eqs. (iii) and (iv), we get

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = \frac{I_P}{I_S}$$

$$\text{or } I_S = I_P \left(\frac{N_P}{N_S} \right) = \frac{I_P}{K} \quad \dots(v)$$

where $K = (N_S/N_P) = \text{transformer ratio}$.

When $K > 1$, then $N_S > N_P$. The transformer is known as step up transformer. So, in a step up transformer, the number of turns in secondary coil is more than the number of turns in primary coil. This transformer converts a low voltage at high current to a high voltage at low current.

When $K > 1$, then $N_s < N_p$. The transformer is known as step-down transformer. So, in a step down transformer, the number of turns in primary coil is more than the number of turns in secondary coil. This transformer converts high voltage at low current to a low voltage at high current.

When a resistance R is introduced in the secondary circuit, then a current I_s flows in it. The current is in phase with the voltage. The current is given by $I_s = E_s/R$

$$\text{where } I_s = I_p \left(\frac{N_p}{N_s} \right) \quad \text{and} \quad E_s = E_p \left(\frac{N_s}{N_p} \right)$$

Putting these values, we get

$$I_p \left(\frac{N_p}{N_s} \right) = E_p \times \left(\frac{N_s}{N_p} \right) \times \frac{1}{R}$$

$$\text{or } I_p = \frac{E_p}{(N_p/N_s)^2 R} = \frac{E_p}{R_{eq.}}$$

$$\text{where } R_{eq.} = \left(\frac{N_p}{N_s} \right)^2 R$$

This current I_p is the same as if we connect a resistance $R_{eq} = (N_p/N_s)^2 R$ across the primary. This effect is called impedance transformation. Impedance matching is an important function of the transformer.

Efficiency of a transformer: There are some losses of energy due to primary resistance, hysteresis in the core, eddy currents in the core, etc., in an ordinary transformer. The efficiency of a transformer is defined as the ratio of output power to the input power

$$\eta = \frac{\text{output power}}{\text{input power}} = \frac{E_s I_s}{E_p I_p}$$

For an ideal transformer, $\eta = 1$ (100 %). Due to transformer, losses, the efficiency is less than one (less than 100 %).

It is important to mention that a transformer is an ac device and it cannot work on dc.

Applications: We know that power is generated at around 11 kV in a power-house. For transmission, we utilize higher ratings, say, around 110 kV or 200 kV. We need step-up transformers for this purpose. Again to supply voltage at load centres, it has to be reduced to, say, 3.6 kV and then to supply it to customers to 220 V. For this we need step-down transformers. This is the major application of a transformer. Here it would not be wrong to say that the wide application of ac, instead of dc, is due to the transformers alone. Since ac can be easily converted from high voltage to low voltage and vice-versa by using a transformer, it has led to widespread development of the ac systems.

ILLUSTRATION 6.16

A power transmission line feeds input power at 2300 V to a step-down transformer with its primary windings having 4000 turns. What should be the number of turns in the secondary in order to get output power at 230 V?

Sol. Here, $\epsilon_p = 2300$ V, $N_p = 4000$, $\epsilon_s = 230$ V
Let N_s be the required number of turns in the secondary.

$$\begin{aligned} \text{As } \frac{\epsilon_s}{\epsilon_p} &= \frac{N_s}{N_p}, N_s = \frac{\epsilon_s}{\epsilon_p} \times N_p \\ &= \frac{230}{2300} \times 4000 = 400 \end{aligned}$$

ILLUSTRATION 6.17

A step-up transformer is used on a 120 V line to provide a potential difference of 2400 V. If the primary has 75 turns, how many turns must the secondary have ?

Sol. Here, voltage across the primary, $\epsilon_p = 120$ V

Number of turns in the primary, $N_p = 75$

Voltage across the secondary, $\epsilon_s = 2400$ V

If N_s is number of turns in the secondary,

$$\begin{aligned} N_s &= N_p \frac{\epsilon_s}{\epsilon_p} \\ &= 75 \times \frac{2400}{120} = 1500 \end{aligned}$$

ILLUSTRATION 6.18

How much current is drawn by the primary of a transformer which steps down 220 V to 22 V to operate a device with an impedance of 220 Ω .

Sol. Here, $\epsilon_p = 220$ V, $\epsilon_s = 22$ V, $Z = 220 \Omega$

If I_s is the current through the secondary,

$$I_s = \frac{\epsilon_s}{Z} = \frac{22}{220} = 0.1 \text{ A}$$

We know that $\epsilon_p I_p = \epsilon_s I_s$

where I_p is the current drawn by the primary.

Thus, $220 \times I_p = 22 \times 0.1$ or $I_p = 0.01$ A

SKIN EFFECT

When a steady current flows through a cylindrical conductor, it is distributed uniformly over the whole cross-section of the conductor. But when an alternating current of high frequency flows through the conductor, the current density is not uniform throughout the cross-sectional area. The current density is more near the surface than inside the conductor. So the high frequency alternating current is confined to the surface layer. The phenomenon is called skin effect. Since the high frequency alternating current does not pass through the entire cross-section, hence the effective resistance of the conductor for ac is much higher, than dc. So in the transmission of ac, a bundle of wire is preferred than a thick wire to reduce the resistance caused by skin effect.

ALTERNATING CURRENT (AC) GENERATOR

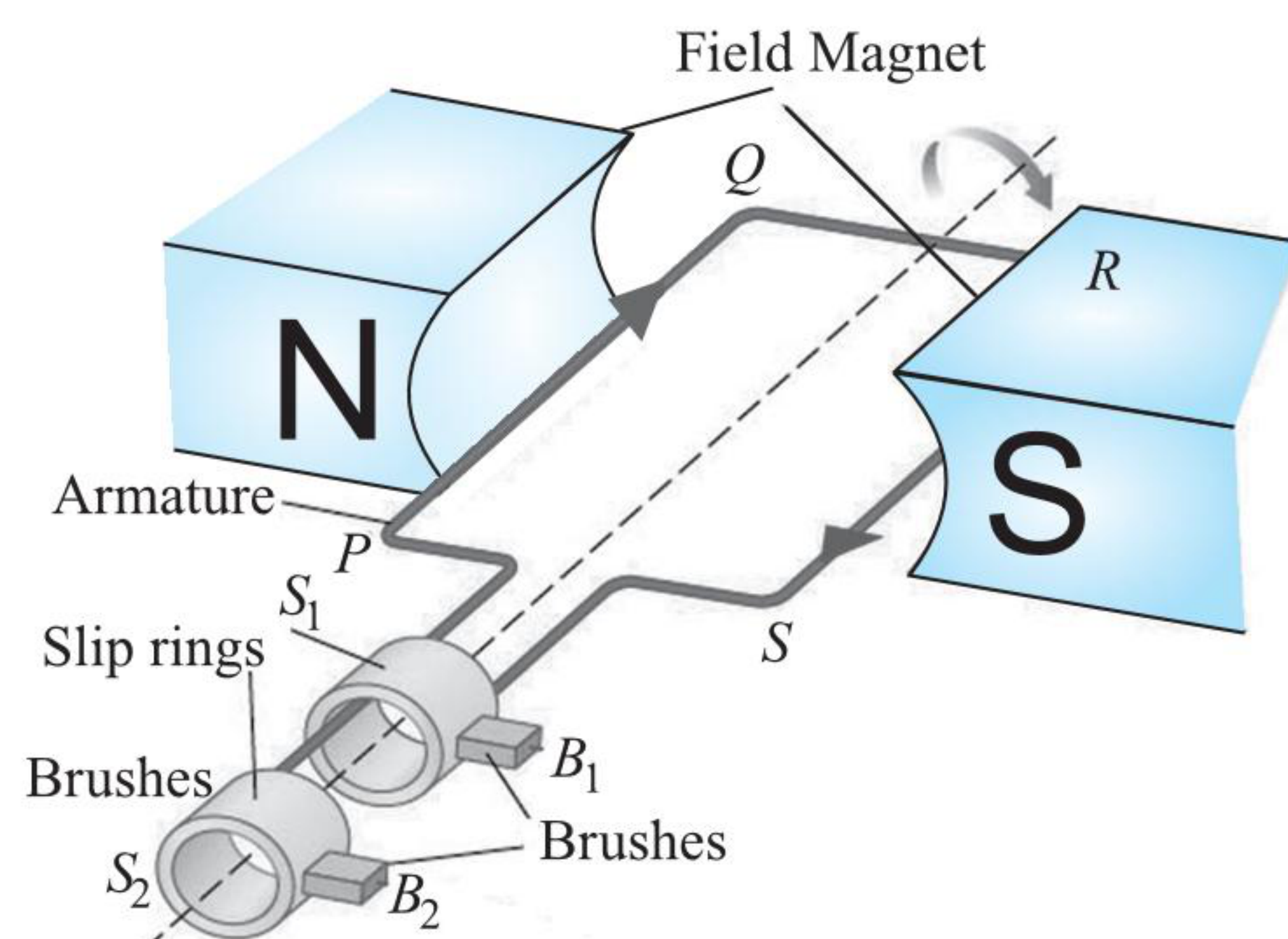
A generator (or a dynamo) is a machine used for generating electric current by converting mechanical energy into electrical energy using electromagnetic induction.

An **a.c. generator** is the one which produces a current that alternates or changes its direction regularly after a fixed interval

of time, i.e., it is a device which converts mechanical energy into alternating form of electrical energy.

Principle: The working of an a.c. generator is based on the principle of *electromagnetic induction*. When a closed coil is rotated in a uniform magnetic field with its axis perpendicular to the magnetic field, the magnetic flux linked with the coil changes and an induced emf and hence a current is set up in it.

Construction: It consists of the following four main parts:



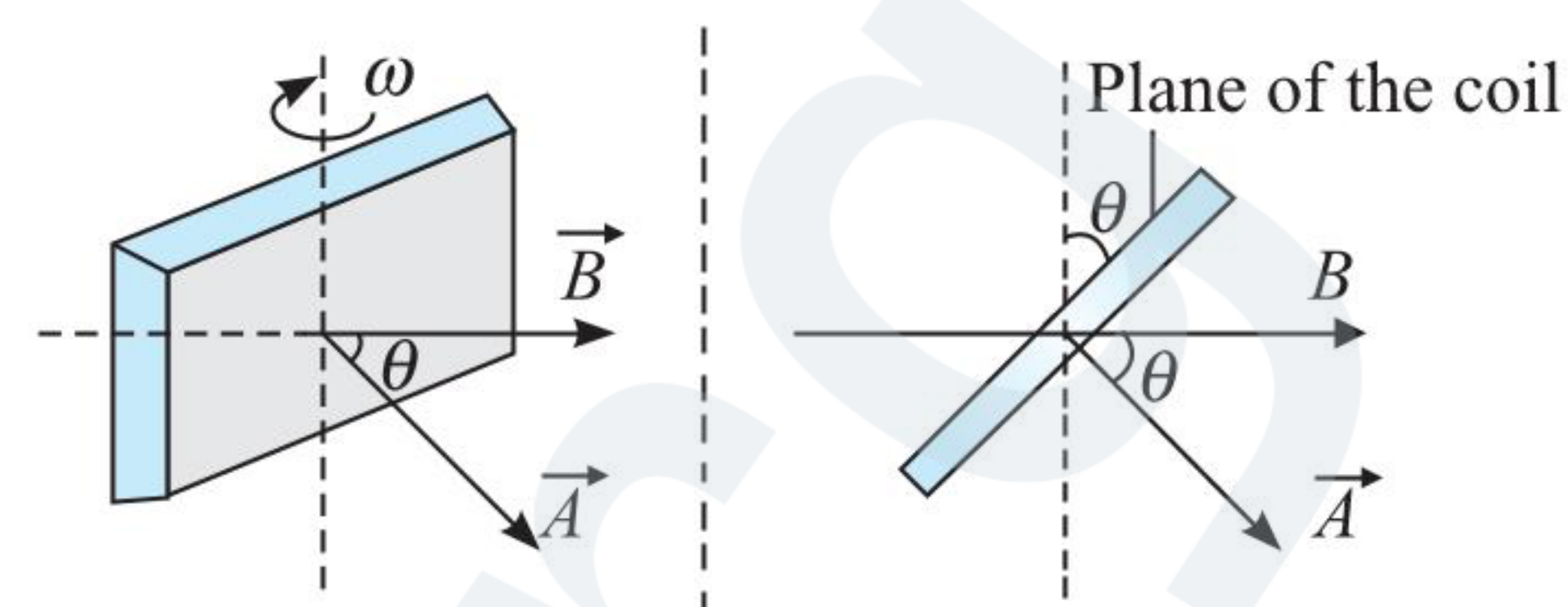
- Field magnet:** It is a permanent magnet of the horseshoe shape in a small dynamo (magneto) or it is a powerful electromagnet in a large dynamo. It produces a strong magnetic field in the region between its pole-pieces.
- Armature:** It consists of a rectangular coil $PQRS$ comprising a large number of turns of insulated copper wire wound over a soft iron core. The armature can be rotated around an axle between the two poles of a strong magnet.
- Slip rings:** The two ends of the armature coil are connected to two coaxial brass rings R_1 and R_2 called slip rings. The rings are rigidly fixed to same shaft which is used to rotate the coil. The slips are insulated from each other as well as from the shaft. As the armature coil rotates, the slip rings also rotate about the same axis of rotation.
- Brushes:** Two graphite or flexible metallic plates B_1 and B_2 are called brushes. B_1 is in constant touch with R_1 and B_2 is in constant touch with R_2 . It is with the help of these brushes that the current is passed on from the armature and the rings to the external circuit of resistance R . Brushes are stationary, i.e., they do not rotate with the rotation of the coil.

Working: As the armature coil rotates, the magnetic flux linked with it changes and so an induced current flows through it. Suppose initially the coil $PQRS$ be in the vertical position and it is rotated in the clockwise direction. The side PQ moves downward and SR moves upward. According to Fleming's right-hand rule, the induced current flows from Q to P and from S to R . So, during the first half rotation of the coil, the induced current flows in the direction $SRQP$, with brush B_2 acting as positive terminal and brush B_1 as negative terminal. During the second half rotation, the side PQ moves upward and SR moves downward. The direction of induced current is reversed, i.e., it flows along $PQRS$, so that the brush B_2 now functions as the positive terminal and brush B_1 as the negative terminal. Thus, the direction of current in the external circuit is reversed after every

half cycle. Hence alternating current is produced by the generator. Such a generator which generates alternating current is called an a.c. generator or an alternator.

EXPRESSION FOR INDUCED EMF IN AC DYNAMO

The emf in the rotating armature of the dynamo can be calculated as follows. In figure, let



N = total number of turns in the armature coil,

A = area of the armature,

B = magnetic field,

ϕ_B = flux linked with the armature at any instant and

θ = angle between $\vec{B}$ and $\vec{A}$ at that instant.

$$\text{Clearly, } \phi_B = N(\vec{B} \cdot \vec{A}) = NBA \cos \theta$$

$$\text{Thus, induced emf, } \varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(NBA \cos \theta)$$

$$\text{or } \varepsilon = -\frac{d\phi_B}{dt} = -\frac{d}{dt}(NBA \cos \theta) \quad \dots(1)$$

ε will be maximum (in magnitude) when

$$\sin \theta = \pm 1 \text{ or } \theta = 90^\circ \text{ or } 270^\circ$$

This is the case when the magnetic field is in the plane of the armature and the rate of change of magnetic flux with respect to time is the maximum.

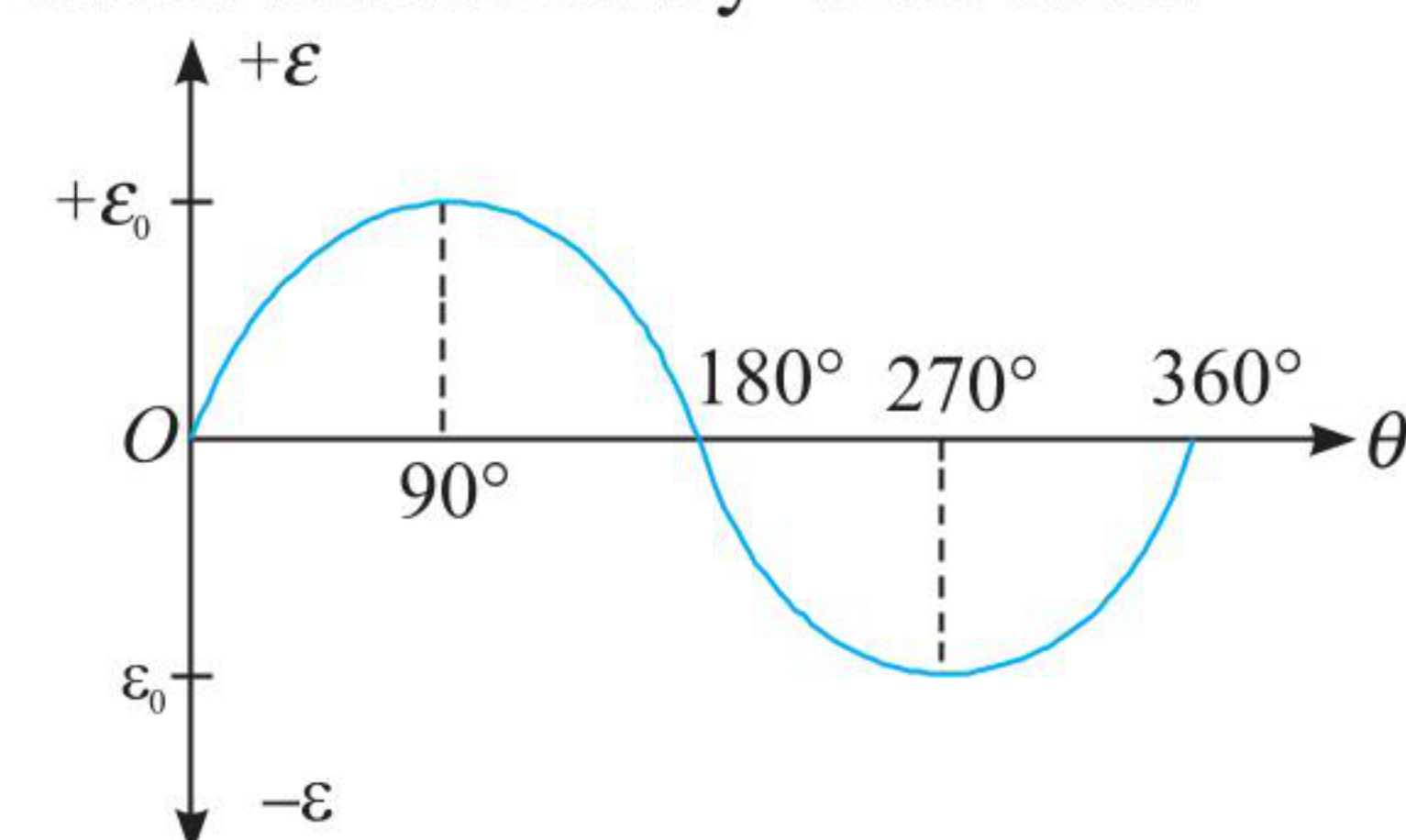
If ε_0 is the maximum value of ε ,

$$\varepsilon_0 = NBA\omega \quad \dots(2)$$

From Eqs. (1) and (2),

$$\varepsilon = \varepsilon_0 \sin \theta = \varepsilon_0 \sin \omega t \quad \dots(3)$$

Further, ε is zero when $\vec{B} \sin \theta = 0$ or $\theta = 0^\circ$ or 180° or 360° . This is the case when $\vec{B}$ is perpendicular to the plane of the armature and the time rate of change of flux is zero. If we plot a graph between ε and θ , we get a sine curve as shown in figure. Thus, the emf varies sinusoidally with time.



Hydroelectric power station: In a hydroelectric power station, water is stored in a dam at a height, from where it falls into giant waterwheels or turbines. These turbines are connected to the loops of wires in a.c. generators. The kinetic energy of the falling water thus gets converted into rotational energy of the turbines and ultimately into electrical energy supplied by the generator.

Thermal power station: In a thermal power station, steam is produced by boiling water using coal or oil as fuel. The turbines coupled to the loops of a.c. generators are rotated by steam rushing past them and thus electrical energy is generated.

Nuclear power plant: In a nuclear power plant, a nuclear fuel is used instead of coal to generate electrical energy.

Important Points:

- **Single phase a.c. generator:** The current produced by rotation of a single coil is known as the single-phase a.c. and the generator is called the single-phase a.c. generator.
- **Two-phase a.c. generator:** In this generator, there are two coils which are at right angles to each other. Each coil has its own pair of slip-rings and brushes. When this arrangement is rotated in a magnetic field; if the emf in one coil is maximum, it will be minimum in the other and vice-versa. Thus, such a generator gives two alternating currents, one of which lags behind the other by 90° . The resultant of both these alternating currents is known as two-phase a.c. and the generator producing it is called two-phase a.c. generator.
- **Three-phase a.c. generator:** In this generator, there are three sets of coils inclined to each other at an angle of 60° . Each set of coil has its own pair of slip rings and brushes. From such an arrangement, we shall obtain three a.c. currents, each lagging behind the other by 60° . The combination of such alternating currents is called three-phase a.c. and the generator producing it is called three-phase a.c. generator.

ILLUSTRATION 6.19

A student peddles a stationary bicycle, the pedals of which are attached to a 100-turn coil of area 0.10 m^2 . The coil rotates at half a revolution per second and it is placed in a uniform magnetic field of 0.01 T perpendicular to the axis of rotation of the coil. What is the maximum voltage generated in the coil?

Sol. Here $N = 100$, $A = 0.10 \text{ m}^2$, $f = 0.5 \text{ Hz}$,
 $B = 0.01 \text{ T}$

The maximum voltage generated in the coil,

$$\begin{aligned}\epsilon_0 &= NBA\omega = NBA \times 2\pi f \\ &= 100 \times 0.01 \times 0.10 \times 2 \times 3.14 \times 0.5 = 0.314 \text{ V}\end{aligned}$$

ILLUSTRATION 6.20

An a.c. generator consists of a coil of 50 turns and area 2.5 m^2 rotating at an angular speed of 60 rad s^{-1} in a uniform magnetic field $B = 0.30 \text{ T}$ between two fixed pole pieces. The resistance of the circuit including that of the coil is 500Ω .

- What is the maximum current drawn from the generator?
- What is the flux through the coil when the current is zero? What is the flux when the current is maximum?
- Would the generator work if the coil were stationary and instead the pole pieces rotated together with the same speed as above?

Sol. We know that $\epsilon = \epsilon_0 \sin \omega t = NBA\omega \sin \omega t$

Here $N = 50$, $A = 2.5 \text{ m}^2$, $\omega = 60 \text{ rad s}^{-1}$

$B = 0.30 \text{ T}$, $R = 500 \Omega$

$$\text{Maximum current, } I_0 = \frac{\epsilon_0}{R} = \frac{NBA\omega}{R}$$

$$= \frac{50 \times 0.30 \times 2.5 \times 60}{500} = 4.5 \text{ A}$$

$$(b) \text{ Current, } I = I_0 \sin \omega t = \frac{NBA\omega}{R} \sin \omega t$$

$$\text{Flux, } \phi_B = NBA \cos \omega t$$

Current is zero if $\sin \omega t = 0$, or $\omega t = 0^\circ$. Then flux is maximum and its value is

$$\begin{aligned}\phi_B &= NBA \cos 0^\circ \\ &= NBA = 50 \times 0.30 \times 2.5 \text{ Wb} = 37.5 \text{ Wb}\end{aligned}$$

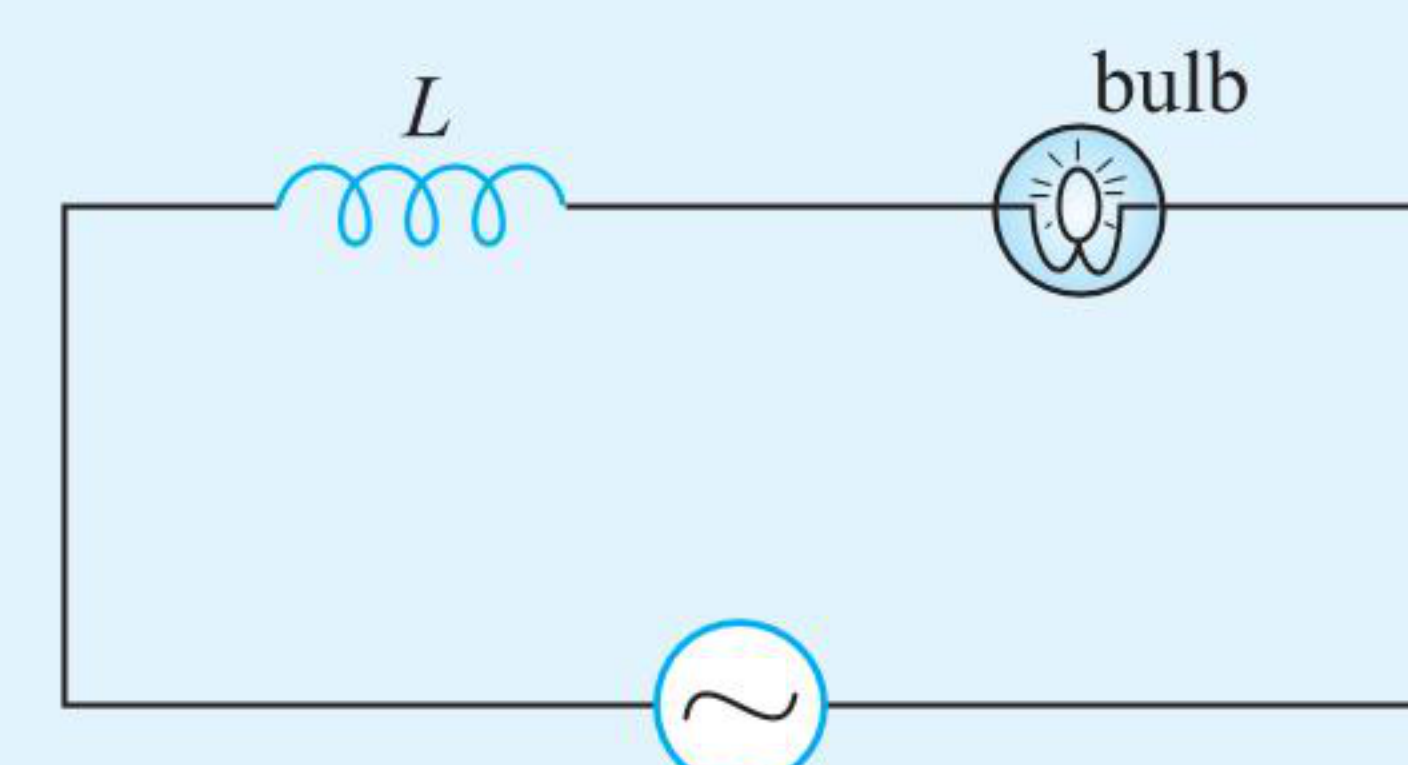
Current is maximum when $\sin \omega t = 1$ or $\omega t = 90^\circ$. Then flux is zero because

$$\phi_B = NBA \cos 90^\circ = 0$$

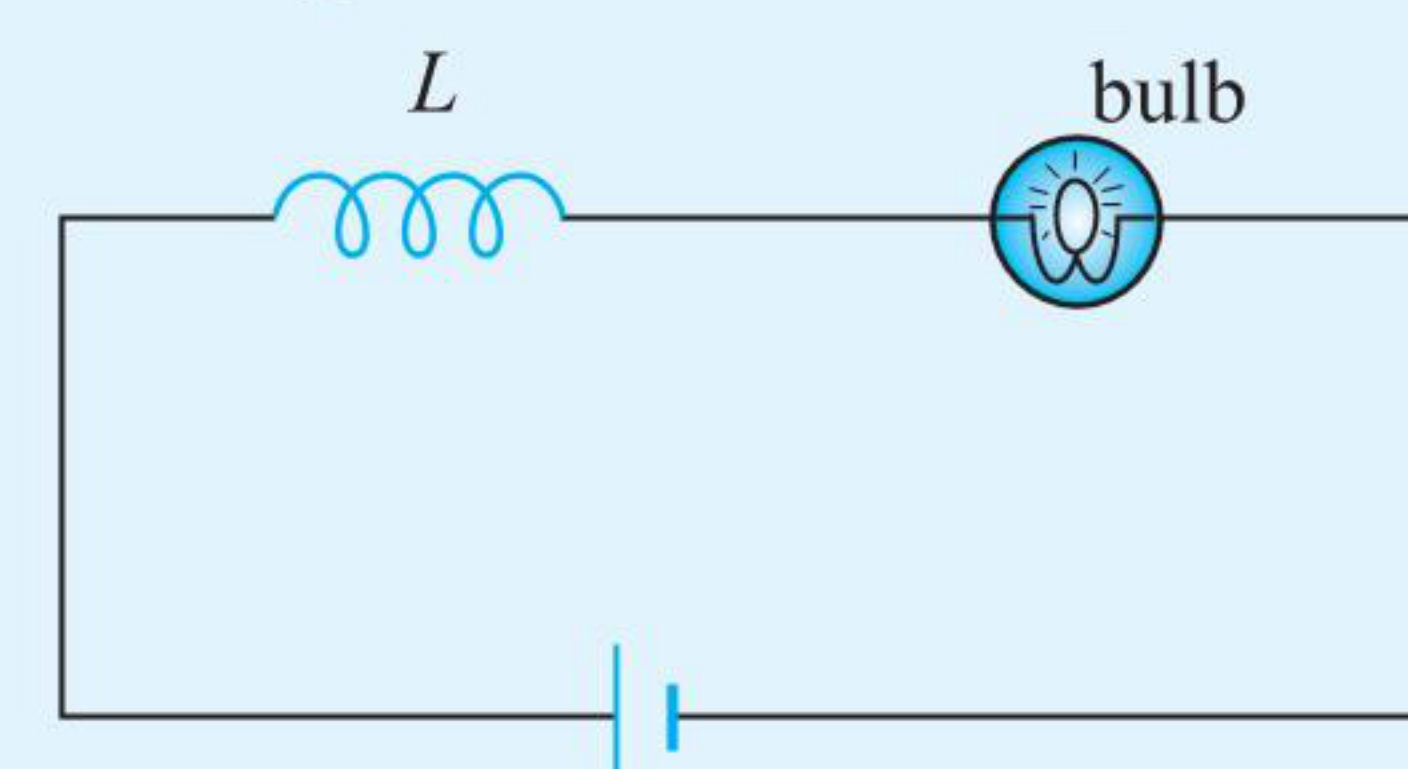
- Yes, the generator would work if the coil were stationary and the pole pieces are rotated together with same speed, because all that we need to set up an induced emf is the relative motion between the coil and the pole pieces. This will also bring about the necessary flux change.

CONCEPT APPLICATION EXERCISE 6.1

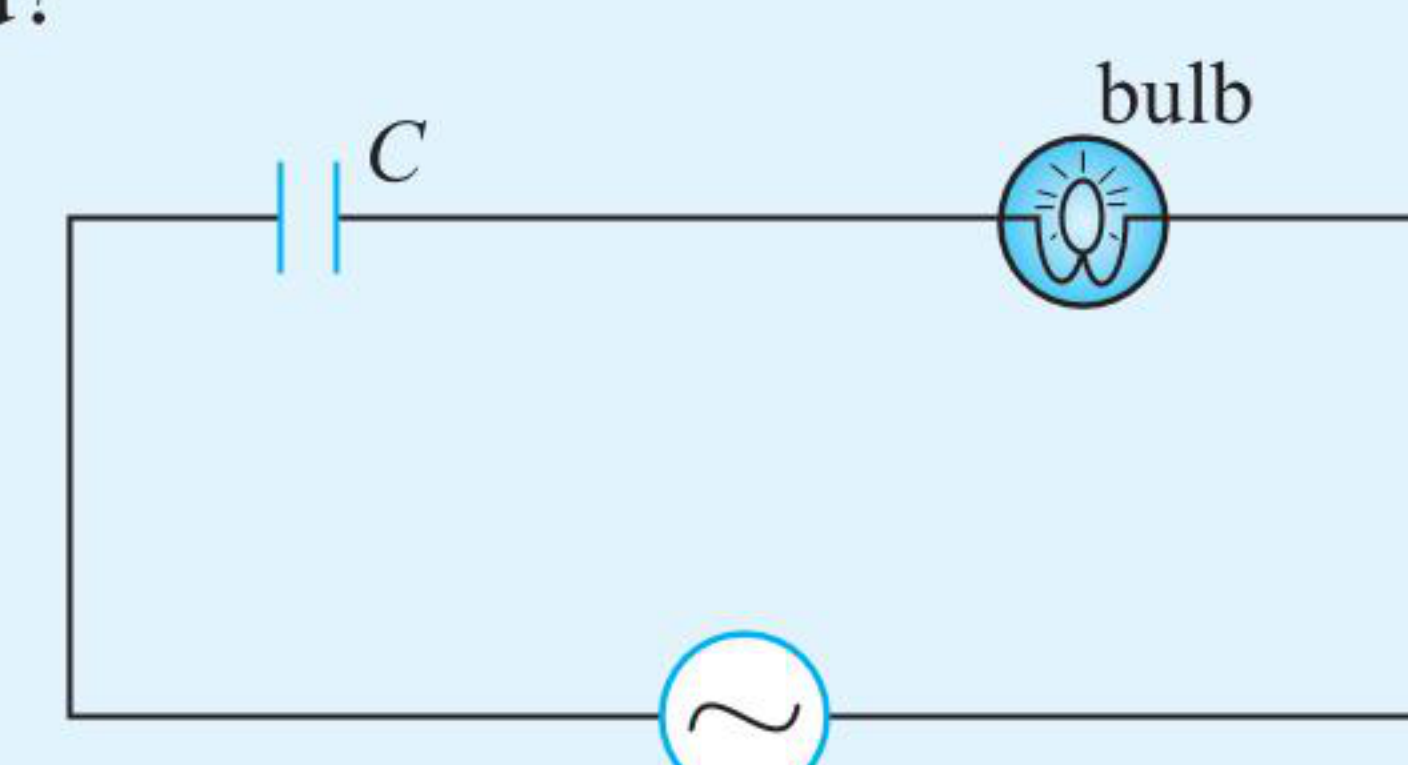
- Can we use 15 Hz ac for lighting purposes?
- A soft iron rod is inserted in the solenoid. After this what will be the effect on the brightness of the bulb?
- In the above question, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



- An iron rod is inserted in the inductor, what will be the effect on the brightness of the bulb?



- In the circuit shown in figure, what will be the effect on the brightness of bulb if frequency of the source ω is increased?



- In the above question, what will be the effect on the brightness of bulb if capacitance C is reduced?

- 7. An inductor wire has some resistance R when connected to dc. It is connected to an ac source. Now the impedance of the wire will be more than R . (True/False)
- 8. The element of an electric heater is in the form of a coil. Once it is heated by dc voltage and then by ac voltage of equal potential difference. Will the production of heat in both cases be same or different?
- 9. When L and C are in series, voltage across them is in phase (True /False). The current in them is in phase (True/ False).
- 10. In a series LCR circuit, the frequency of the source is more than resonance frequency. The current in the circuit leads the voltage in phase (True/False).
- 11. In a series LCR circuit, voltage drop across L can be more than applied voltage (True/False).

- 12. For circuits used for transporting electric power, a low power factor implies larger power loss in transmission. Explain?
- 13. At very high frequency of ac, capacitor behaves like a conductor. (True/False]
- 14. In a series LCR circuit, at resonance, power factor is _____.
- 15. Can we have resonance in LR or CR circuits?

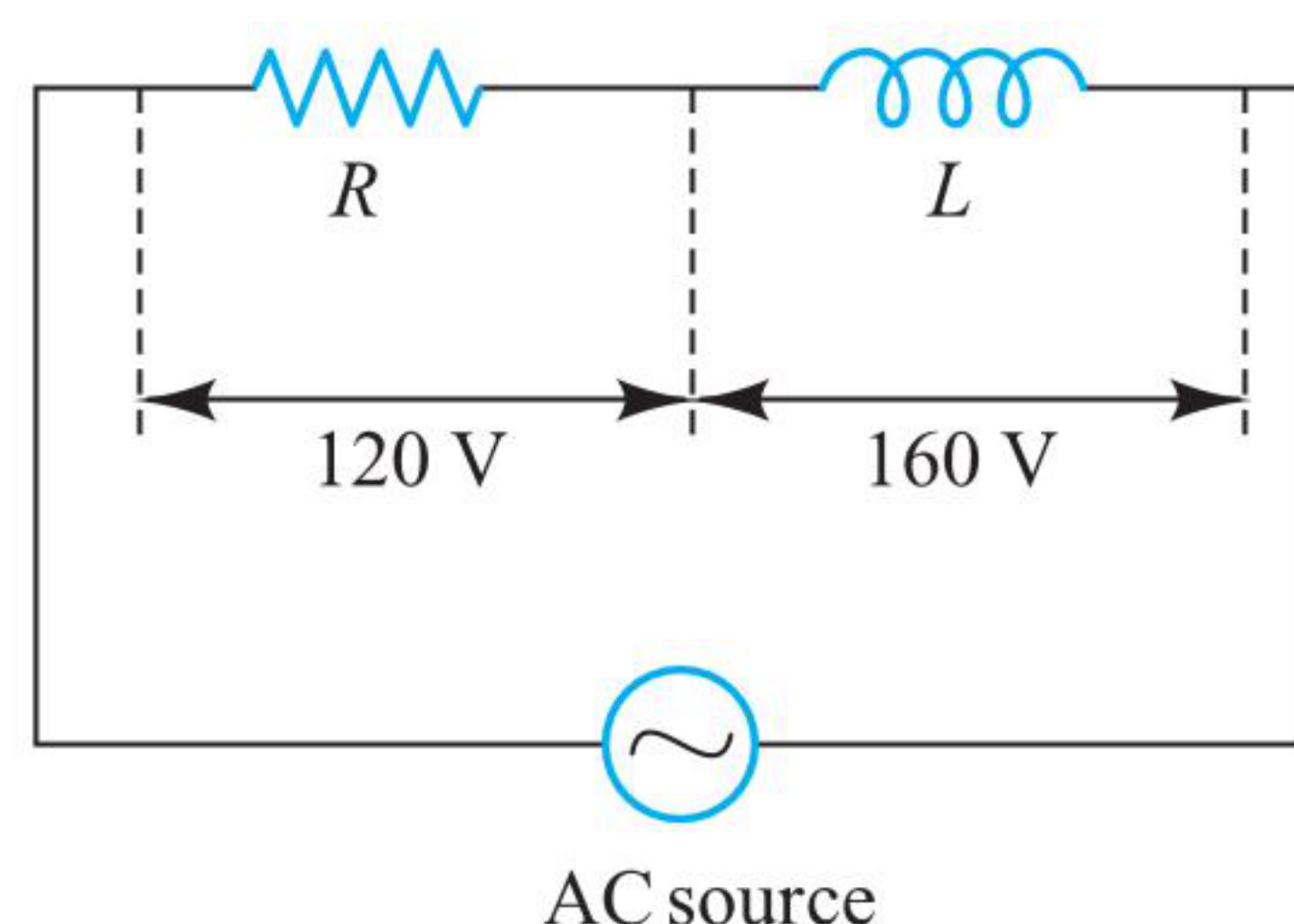
ANSWERS

- | | |
|--------------------------------------|-----------------------------|
| 1. Yes | 2. Brightness will decrease |
| 3. Brightness will decrease | |
| 4. First decrease, then becomes same | |
| 5. Brightness will increase | 6. Brightness will decrease |
| 7. True | 8. Different |
| 9. False, True | 10. False |
| 11. True | 13. True |
| 14. 1 | 15. No |

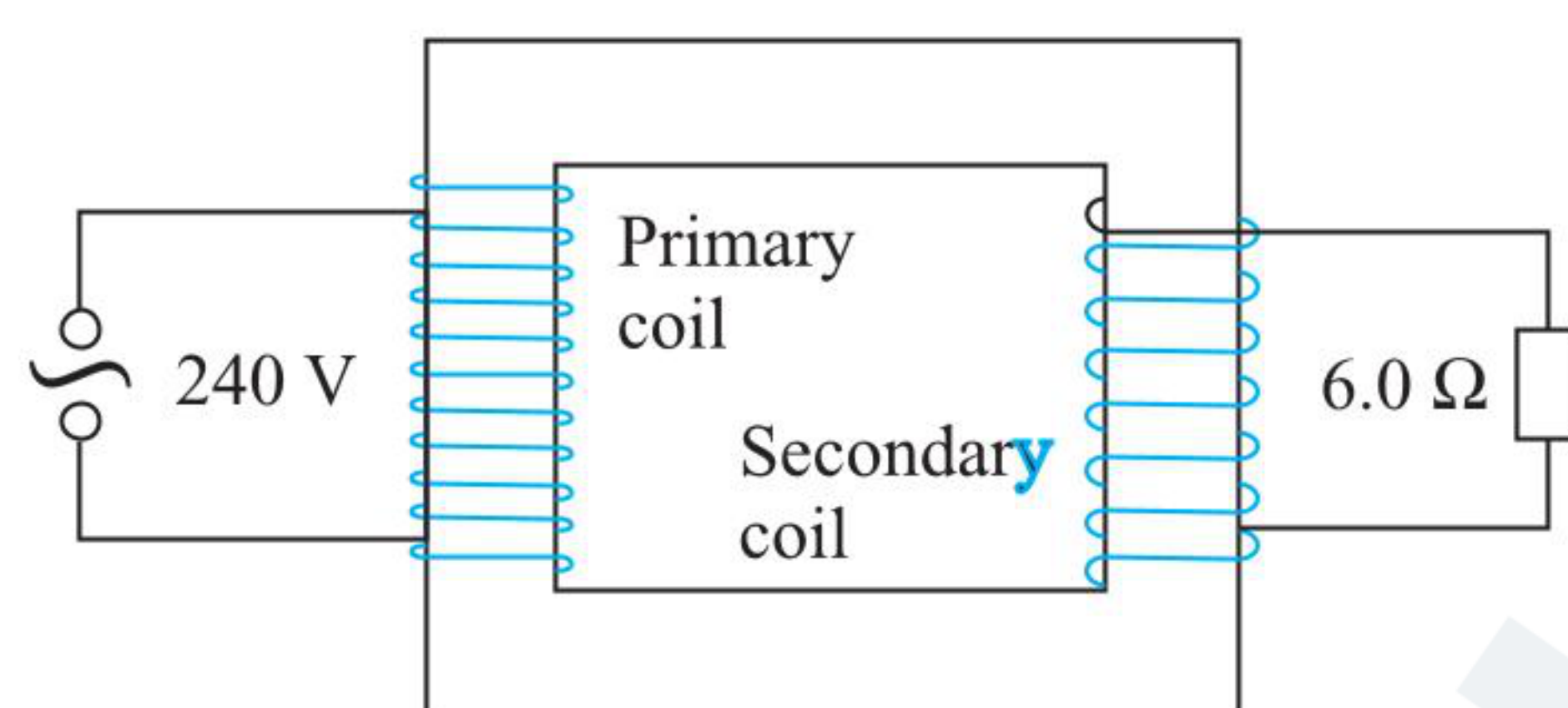
Exercises

Single Correct Answer Type

1. The circuit given in figure has a resistanceless choke coil L and a resistance R . The voltages across R and L are also given in the figure. The virtual value of the applied voltage is

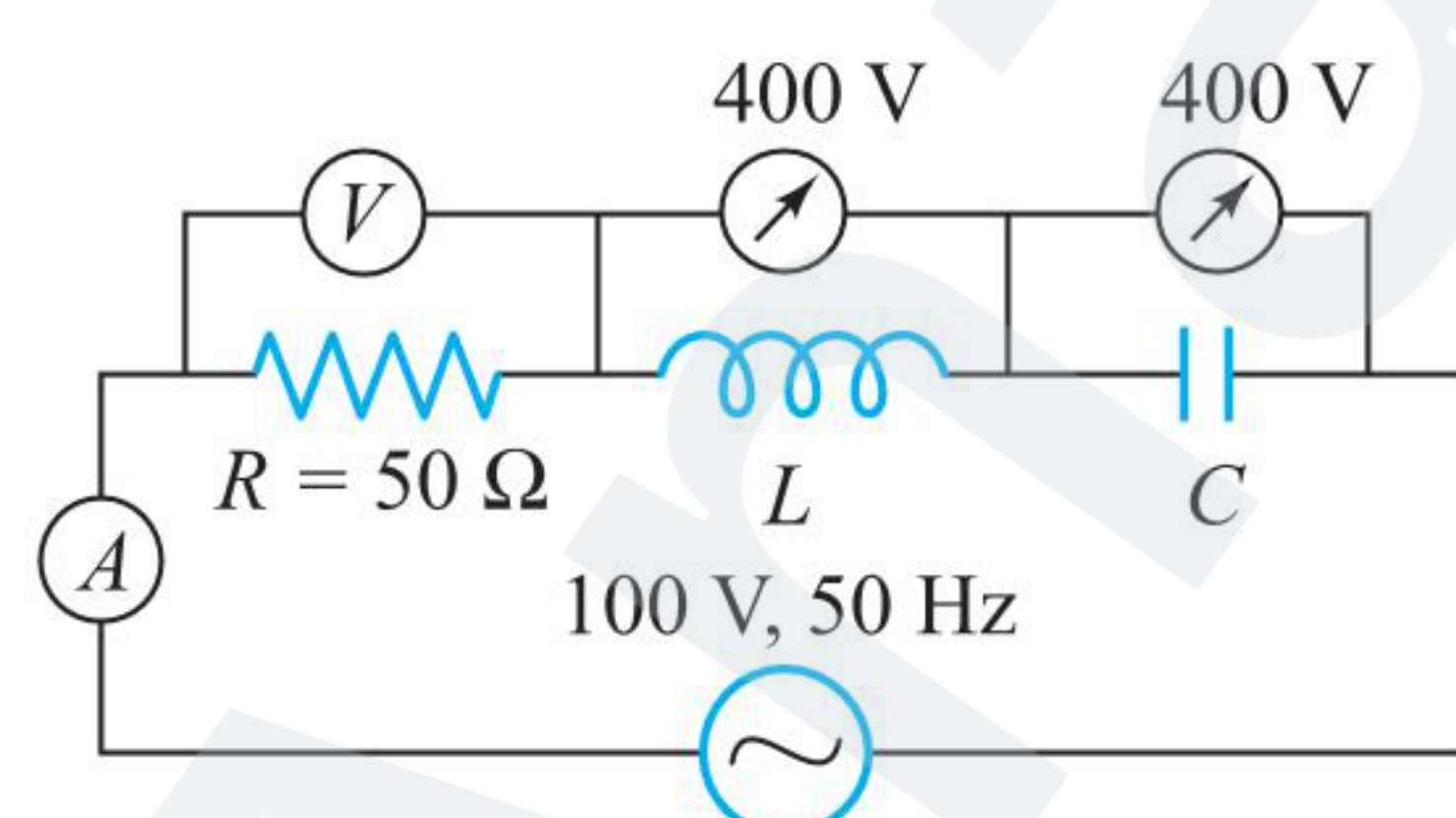


- (1) 100 V (2) 200 V
(3) 300 V (4) 400 V
2. Figure shows an iron-cored transformer assumed to be 100% efficient. The ratio of the secondary turns to the primary turns is 1:20.



A 240 V ac supply is connected to the primary coil and a $6\ \Omega$ resistor is connected to the secondary coil. What is the current in the primary coil?

- (1) 0.10 A (2) 0.14 A
(3) 2 A (4) 40 A
3. In the series LCR circuit (figure), the voltmeter and ammeter readings are:

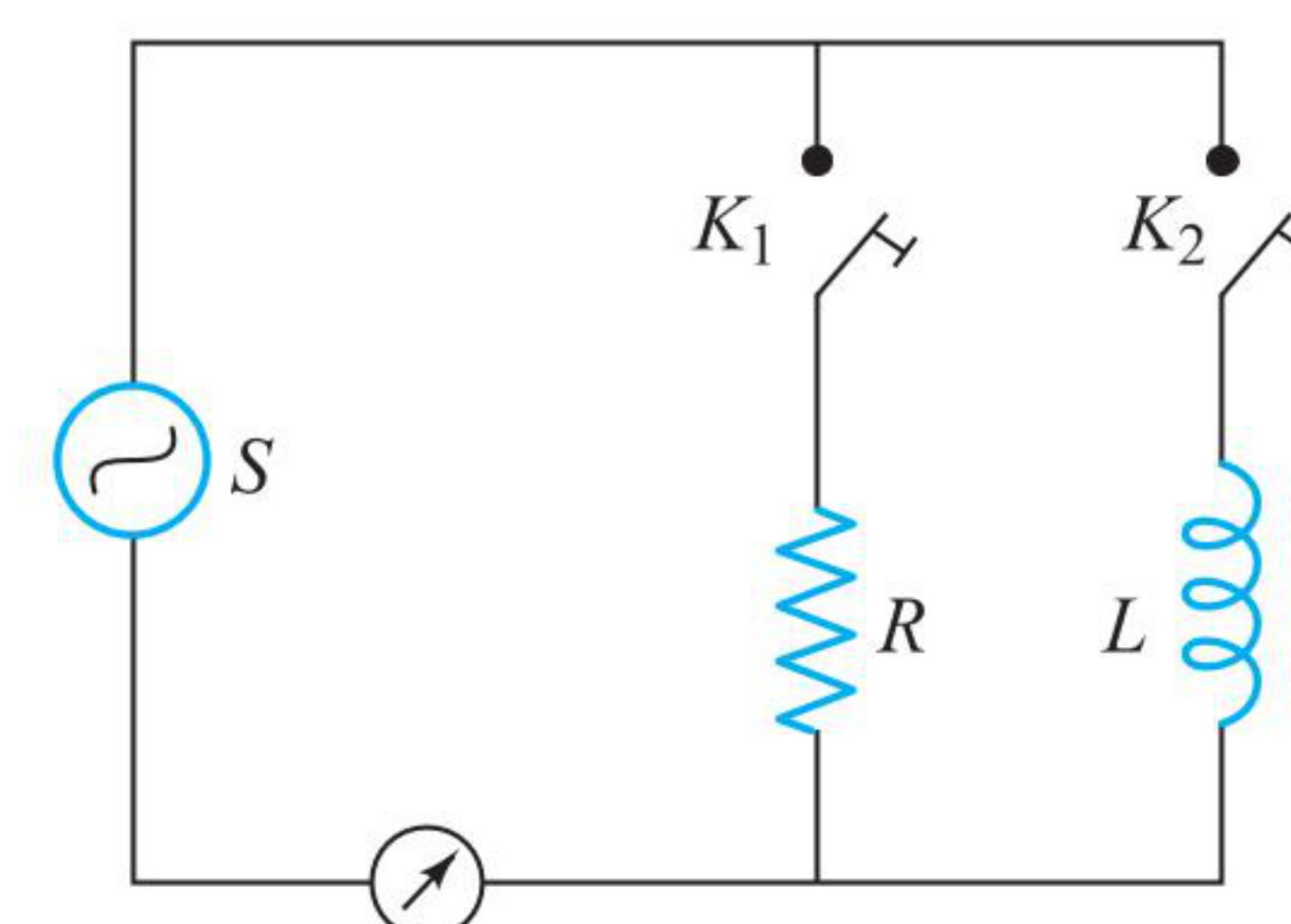


- (1) $V = 100\text{ V}, I = 2\text{ A}$ (2) $V = 100\text{ V}, I = 5\text{ A}$
(3) $V = 1000\text{ V}, I = 2\text{ A}$ (4) $V = 300\text{ V}, I = 1\text{ A}$
4. An LCR circuit contains resistance of $100\ \Omega$ and a supply of 200 V at 300 rad angular frequency. If only capacitance is taken out from the circuit and the rest of the circuit is joined, current lags behind the voltage by 60° . If, on the other hand, only inductor is taken out, the current leads by 60° with the applied voltage. The current flowing in the circuit is
- (1) 1 A (2) 1.5 A
(3) 2 A (4) 2.5 A
5. The peak value of an alternating emf E given by

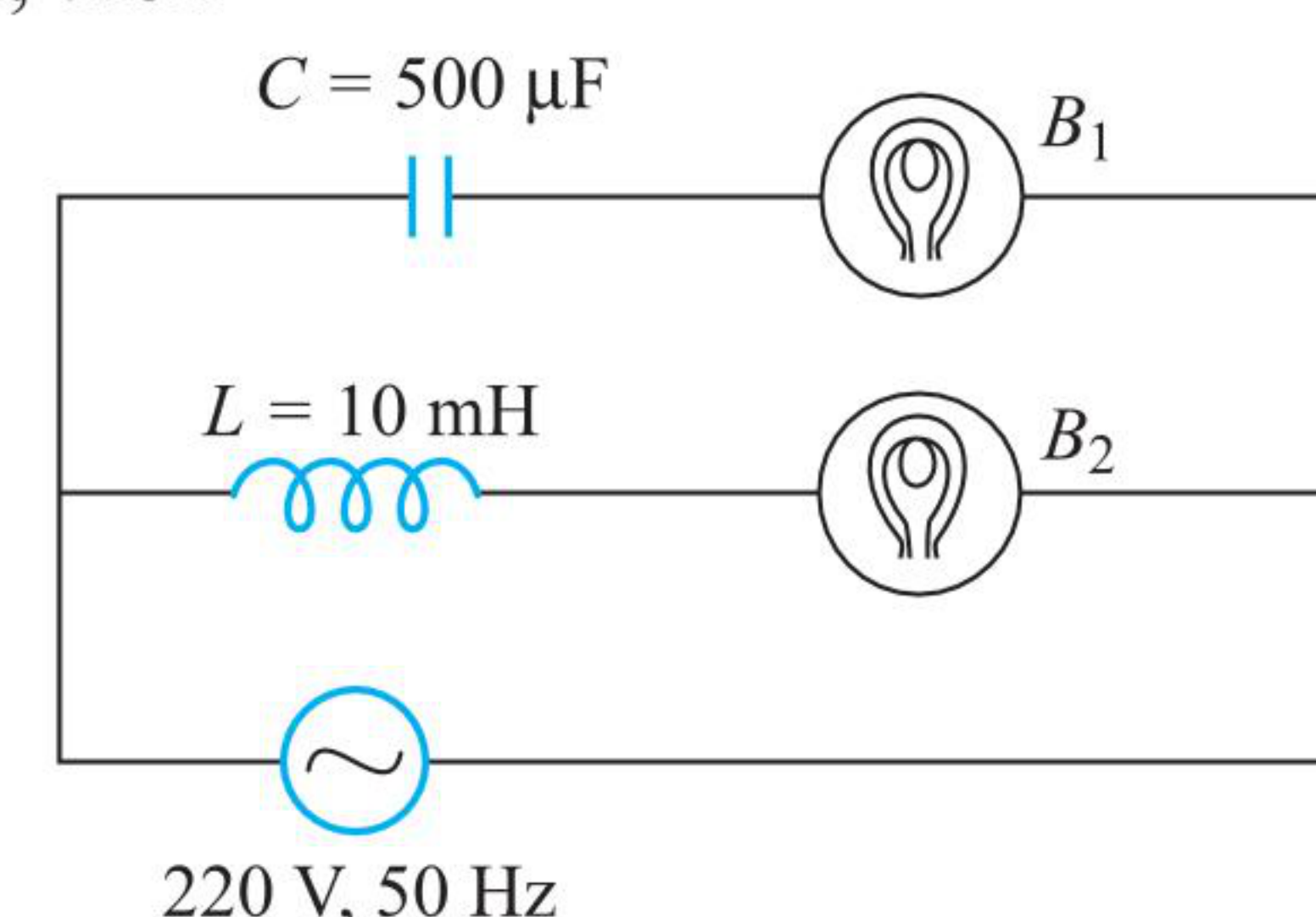
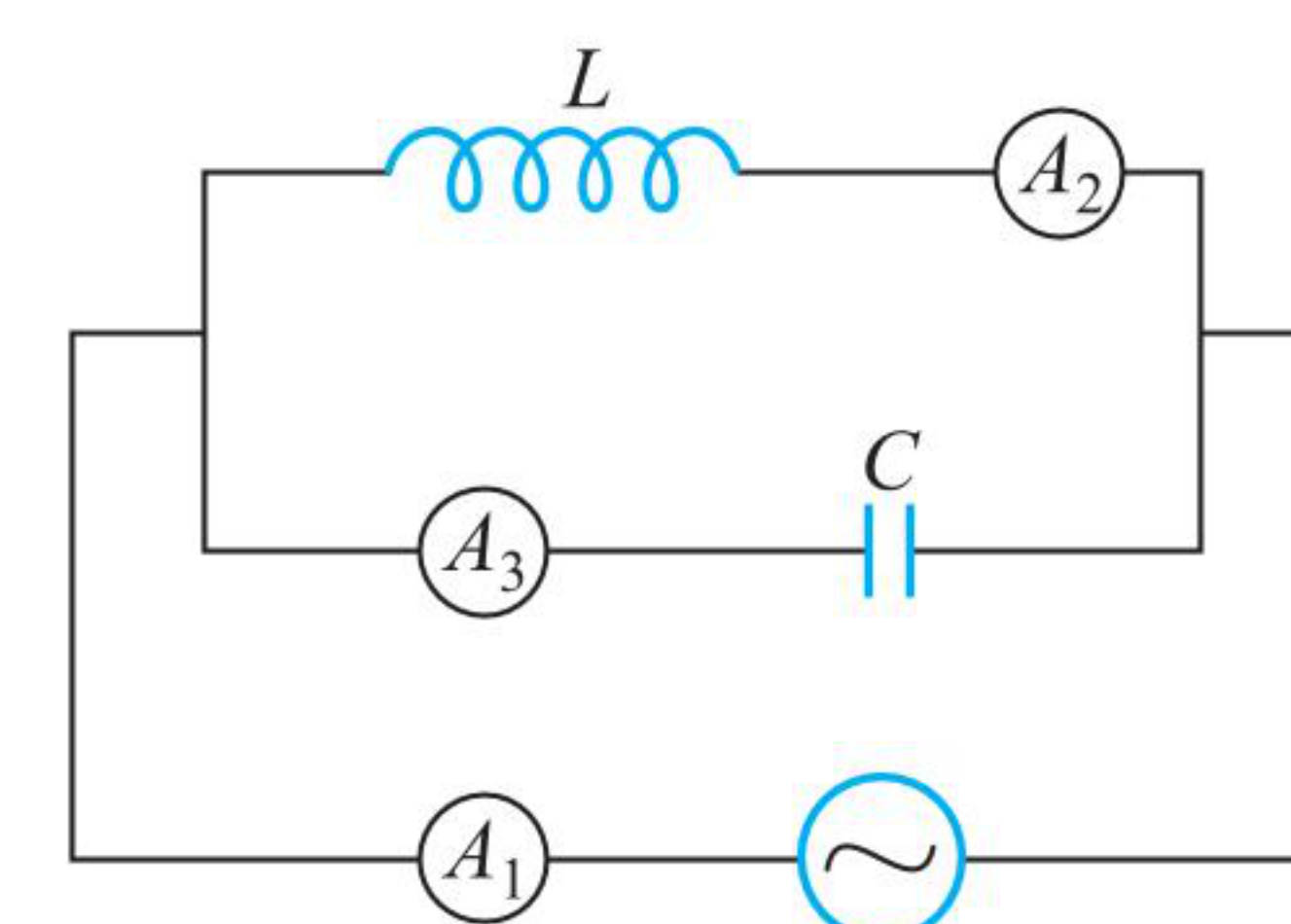
$$E = E_0 \cos \omega t$$

is 10 V and frequency is 50 Hz. At time $t = (1/600)\text{ s}$, the instantaneous value of emf is

- (1) 10 V (2) $5\sqrt{3}\text{ V}$
(3) 5 V (4) 1 V
6. A coil has an inductance of 0.7 H and is joined in series with a resistance of $220\ \Omega$. When an alternating emf of 220 V at 50 cps is applied to it, then the wattless component of the current in the circuit is (take $0.7\pi = 2.2$)
- (1) 5 A (2) 0.5 A
(3) 0.7 A (4) 7 A
7. An inductive coil has resistance of $100\ \Omega$. When an ac signal of frequency 1000 Hz is fed to the coil, the applied voltage leads the current by 45° . What is the inductance of the coil?
- (1) 2 mH (2) 3.3 mH
(3) 16 mH (4) $\sqrt{5}\text{ mH}$
8. In the circuit shown in figure, R is a pure resistor, L is an inductor of negligible resistance (as compared to R), S is a 100 V, 50 Hz ac source of negligible resistance. With either key K_1 alone or K_2 alone closed, the current is I_0 . If the source is changed to 100 V, 100 Hz the current with K_1 alone closed and with K_2 alone closed will be, respectively,

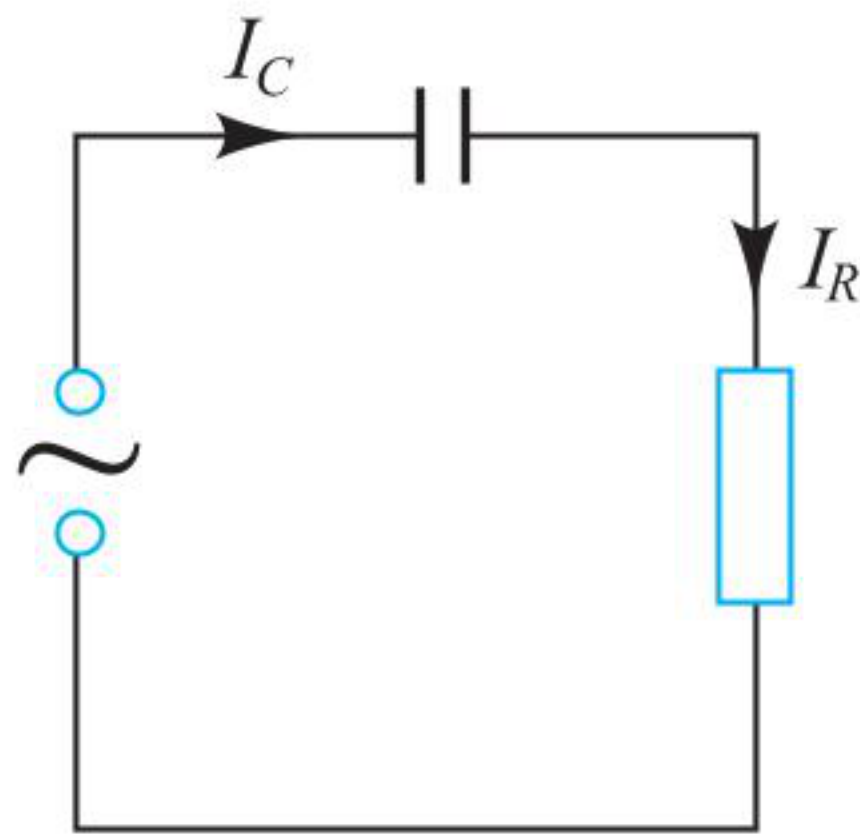


- (1) $I_0, \frac{I_0}{2}$ (2) $I_0, 2I_0$
(3) $2I_0, I_0$ (4) $2I_0, \frac{I_0}{2}$
9. For the circuit shown in figure, the ammeter A_2 reads 1.6 A and ammeter A_3 reads 0.4 A. Then
- (1) $\omega = \frac{4}{\sqrt{LC}}$
(2) $f = \frac{2\pi}{\sqrt{LC}}$
(3) the ammeter A_1 reads 1.2 A
(4) the ammeter A_1 reads 2 A
10. In the circuit shown in figure, if both the bulbs B_1 and B_2 are identical, then



- (1) their brightness will be the same
 (2) B_2 will be brighter than B_1
 (3) B_1 will be brighter than B_2
 (4) only B_2 will glow because the capacitor has infinite impedance

11. Figure shows a source of alternating voltage connected to a capacitor and a resistor. Which of the following phasor diagrams correctly describes the phase relationship between I_c , the current between the source and the capacitor, and I_R , the current in the resistor?



- (1) (2) (3) (4)

12. A sinusoidal alternating current of peak value I_0 passes through a heater of resistance R . What is the mean power output of the heater?

- (1) $I_0^2 R$ (2) $\frac{I_0^2 R}{2}$
 (3) $2I_0^2 R$ (4) $\sqrt{2} I_0^2 R$

13. A resistance of 20Ω is connected to a source of an alternating potential $V = 220 \sin(100 \pi t)$. The time taken by the current to change from the peak value to rms value, is

- (1) 0.2 s (2) 0.25 s
 (3) 2.5×10^{-3} s (4) 2.5×10^{-3} s

14. In LCR circuit current resonant frequency is 600 Hz and half power points are at 650 and 550 Hz. The quality factor is

- (1) $\frac{1}{6}$ (2) $\frac{1}{3}$
 (3) 6 (4) 3

15. An ac voltage is represented by

$$E = 220 \sqrt{2} \cos(50\pi)t$$

How many times will the current become zero in 1 s?

- (1) 50 times (2) 100 times
 (3) 30 times (4) 25 times

16. A transmitter transmits at a wavelength of 300 m. A condenser of capacitance $2.4 \mu\text{F}$ is being used. The value of the inductance for the resonant circuit is approximately

- (1) 10^{-4} H (2) 10^{-6} H
 (3) 10^{-8} H (4) 10^{-10} H

17. A capacitor of capacitance $1 \mu\text{F}$ is charged to a potential of 1 V. It is connected in parallel to an inductor of inductance 10^{-3} H. The maximum current that will flow in the circuit has the value

- (1) $\sqrt{1000}$ mA (2) 1 mA
 (3) 1 μA (4) 1000 mA

18. Using an ac voltmeter, the potential difference in the electrical line in a house is read to be 234 V. If the line frequency is known to be 50 cycles per second, the equation for the line voltage is

- (1) $V = 165 \sin(100 \pi t)$ (2) $V = 331 \sin(100 \pi t)$
 (3) $V = 220 \sin(100 \pi t)$ (4) $V = 440 \sin(100 \pi t)$

19. An inductor and a resistor are connected in series with an ac source. In this circuit.

- (1) the current and the PD across the resistance lead the PD across the inductance
 (2) the current and the PD across the resistance lag behind the PD across the inductance by an angle $\pi/2$
 (3) the current and the PD across the resistance lag behind the PD across the inductance by an angle π
 (4) the PD across the resistance lags behind the PD across the inductance by an angle $\pi/2$ but the current in resistance leads the PD across the inductance by $\pi/2$

20. In a series LCR circuit, the voltage across the resistance, capacitance and inductance is 10 V each. If the capacitance is short circuited, the voltage across the inductance will be

- (1) 10 V (2) $10/\sqrt{2}$ V
 (3) $(10/3)$ V (4) 20 V

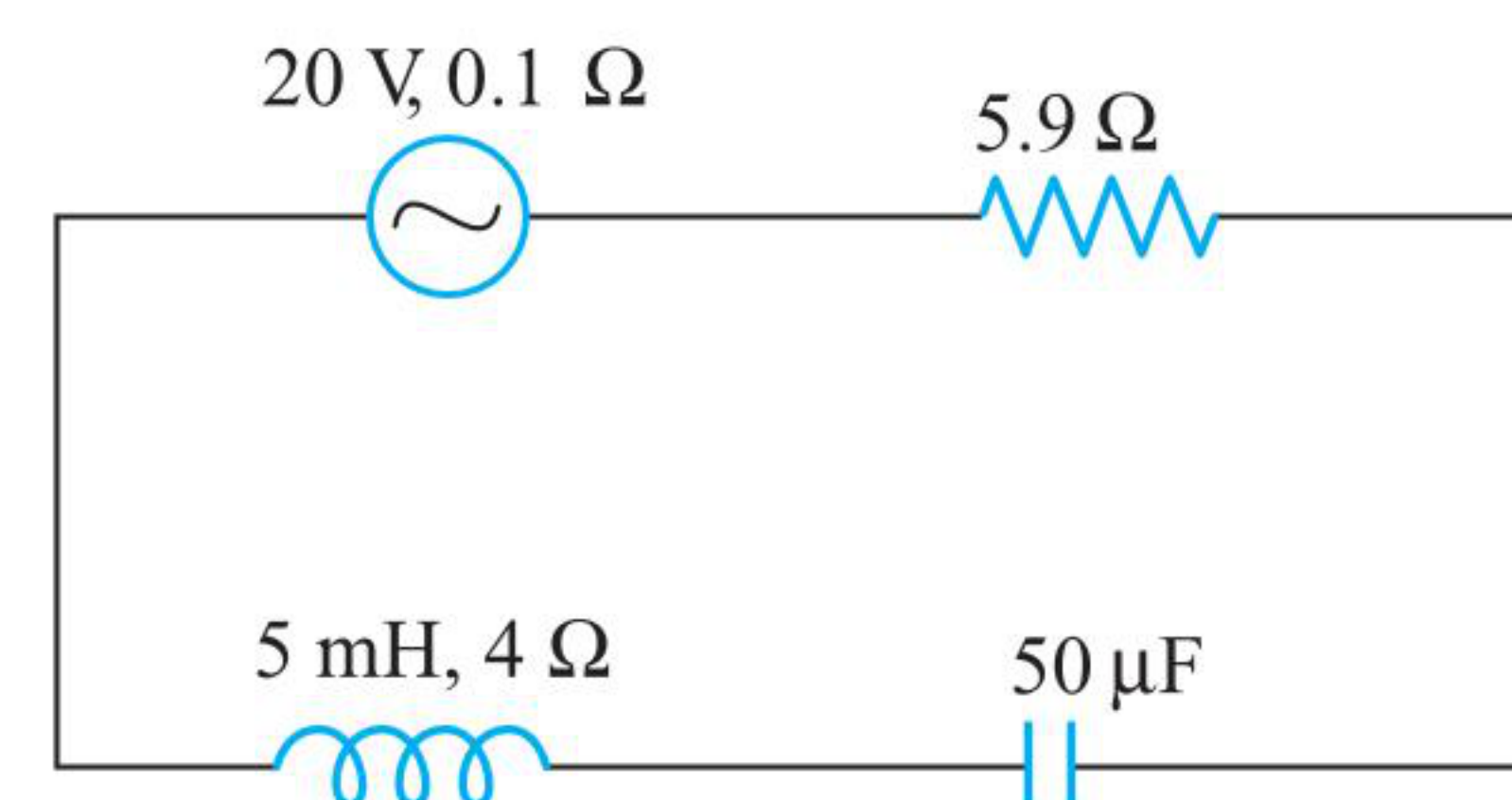
21. An ideal choke takes a current of 10 A when connected to an ac supply of 125 V and 50 Hz. A pure resistor under the same conditions takes a current of 12.5 A. If the two are connected to an ac supply of 100 V and 40 Hz, then the current in series combination of above resistor and inductor is

- (1) $10/\sqrt{2}$ A (2) 12.5 A
 (3) 20 A (4) 10 A

22. A direct current of 5 A is superimposed on an alternating current $I = 10 \sin \omega t$ flowing through a wire. The effective value of the resulting current will be

- (1) $(15/2)$ A (2) $5\sqrt{3}$ A
 (3) $5\sqrt{5}$ A (4) 15 A

23. In the circuit of figure, the source frequency is $\omega = 2000 \text{ rad s}^{-1}$. The current in the circuit will be

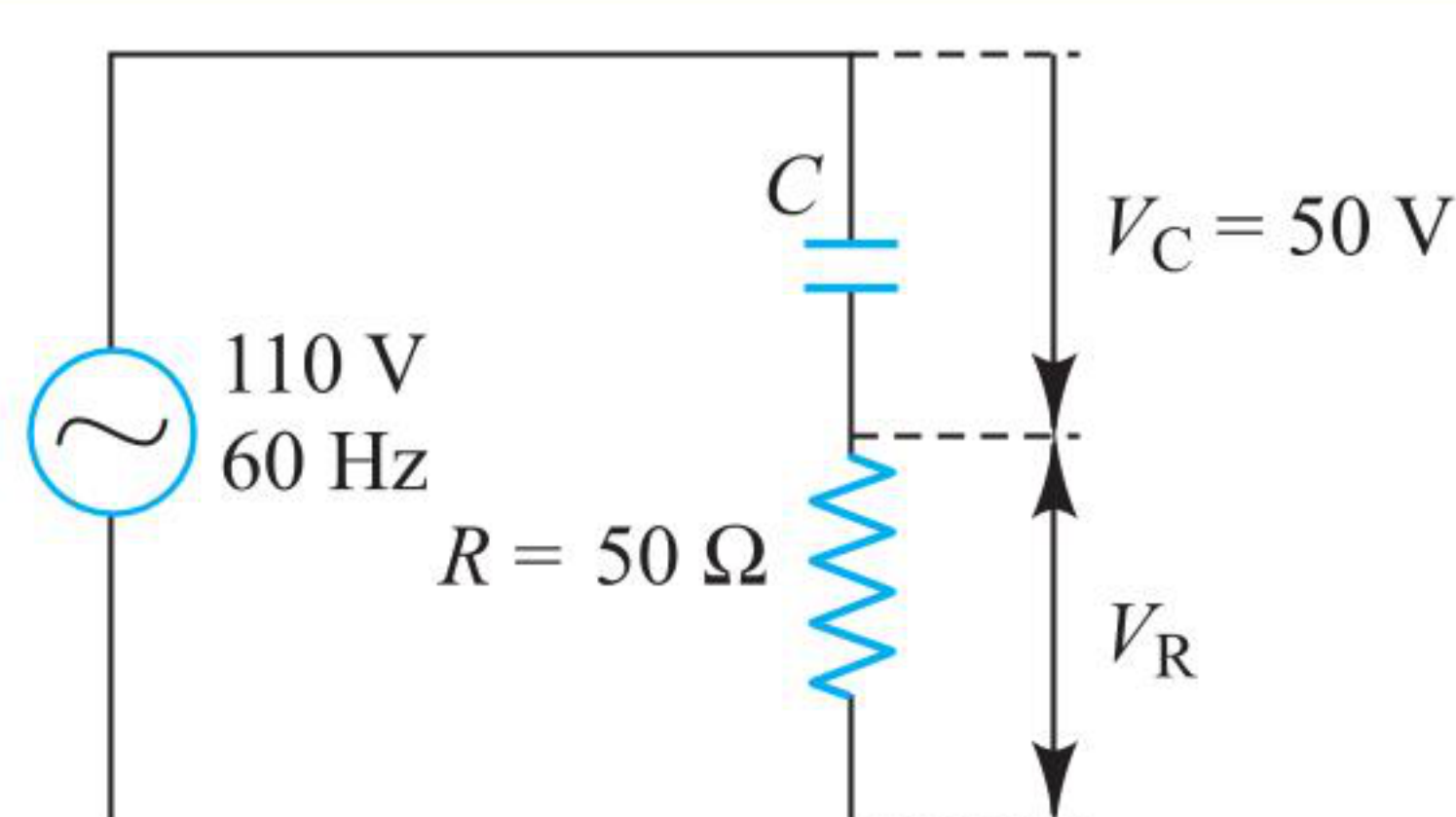


- (1) 2 A (2) 3.3 A
 (3) $2/\sqrt{5}$ A (4) $\sqrt{5}$ A

24. An rms voltage of 110 V is applied across a series circuit having a resistance 11Ω and an impedance 22Ω . The power consumed is

- (1) 275 W (2) 366 W
 (3) 550 W (4) 1100 W

25. In the circuit given in figure, $V_C = 50$ V and $R = 50 \Omega$. The values of C and V_R are



- (1) 3.3 mF, 60 V (2) 104 μF, 98 V
(3) 52 μF, 98 V (4) 2 μF, 60 V

26. An 8 μF capacitor is connected across a 220 V, 50 Hz line. What is the peak value of charge through the capacitor?

- (1) 2.5×10^{-3} C (2) 2.5×10^{-4} C
(3) 5×10^{-5} C (4) 7.5×10^{-2} C

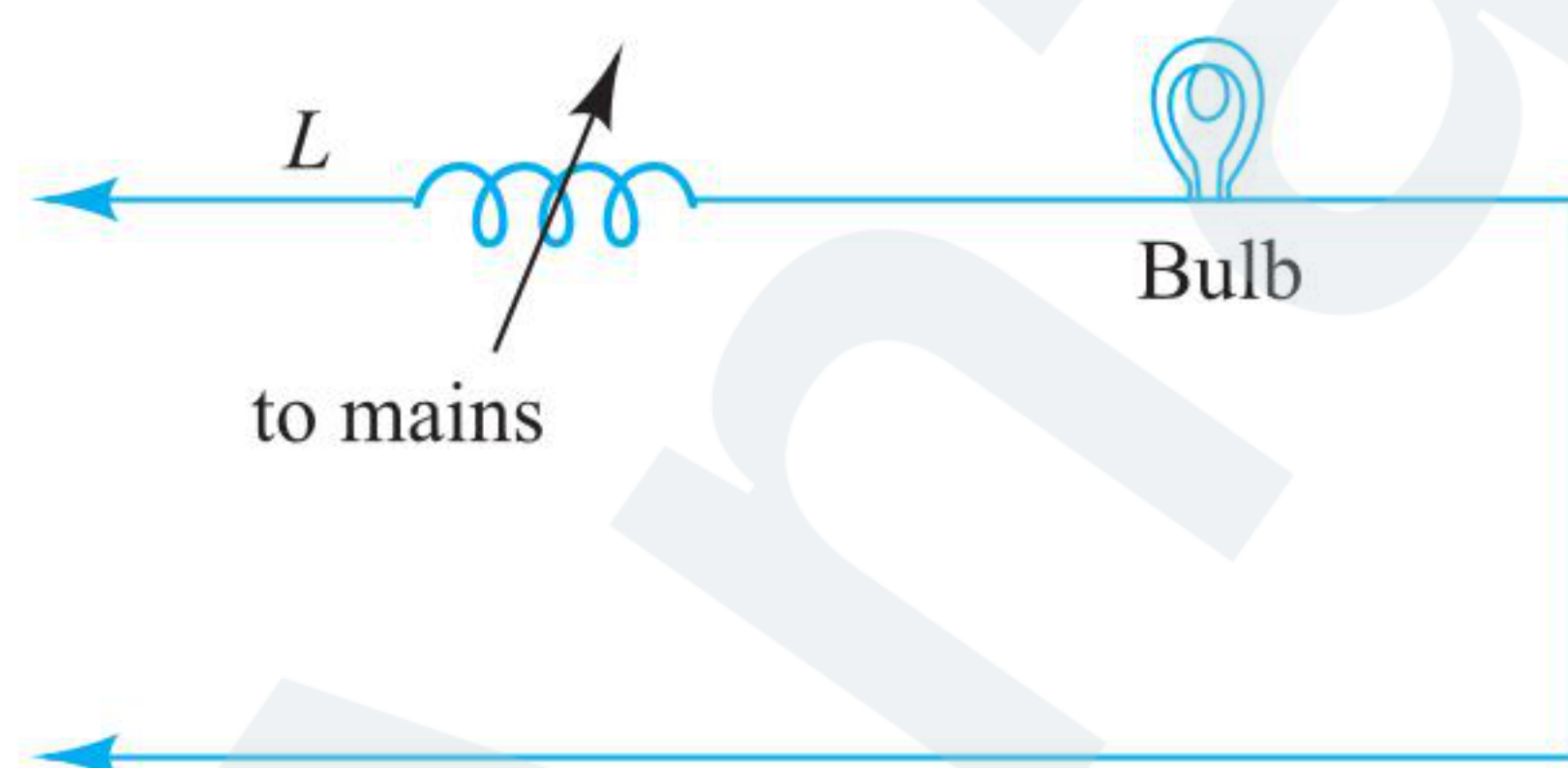
27. A dc ammeter and a hot wire ammeter are connected to a circuit in series. When a direct current is passed through circuit, the dc ammeter shows 6 A. When ac current flows through circuit, the ac ammeter shows 8 A. What will be reading of each ammeter if dc and ac currents flow simultaneously through the circuit?

- (1) dc = 6 A, ac = 10 A (2) dc = 3 A, ac = 5 A
(3) dc = 5 A, ac = 8 A (4) dc = 2 A, ac = 3 A

28. An ac is given by equation $I = I_1 \cos \omega t + I_2 \sin \omega t$. The rms value of current is given by

- (1) $\frac{I_1 + I_2}{2}$ (2) $\frac{(I_1 + I_2)^2}{\sqrt{2}}$
(3) $\sqrt{\frac{I_1^2 + I_2^2}{2}}$ (4) $\frac{I_1^2 + I_2^2}{2}$

29. A typical light dimmer used to dim the stage lights in a theatre consists of a variable induction for L (where inductance is adjustable between zero and $L_{\max}$) connected in series with a light bulb B as shown in figure. The mains electrical supply is 220 V at 50 Hz, the light bulb is rated at 220 V, 1100 W. What $L_{\max}$ is required if the rate of energy dissipated in the light bulb is to be varied by a factor of 5 from its upper limit of 1100 W?

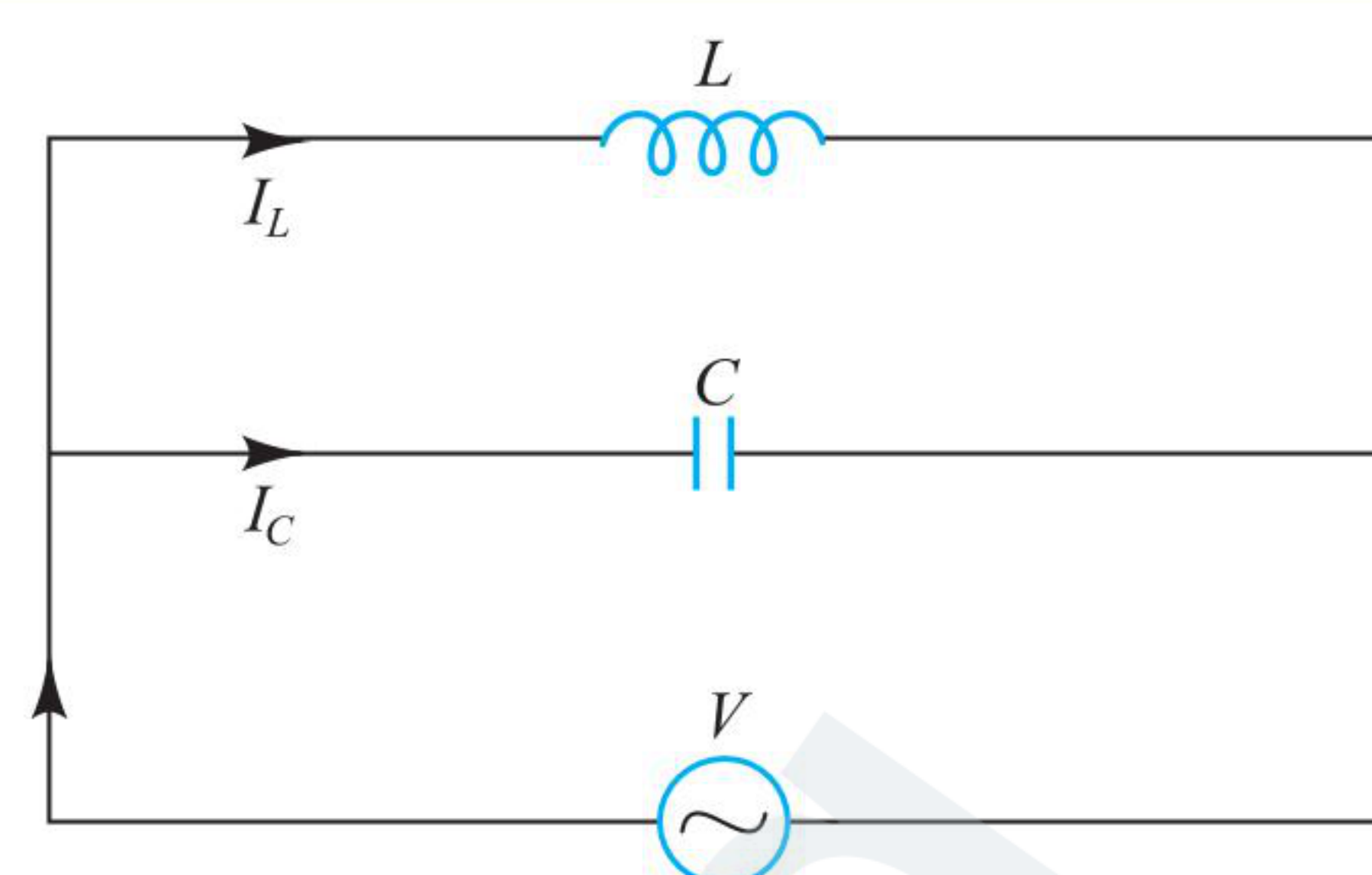


- (1) 0.69 H (2) 0.28 H
(3) 0.38 H (4) 0.56 H

30. Two alternating voltage generators produce emfs of the same amplitude E_0 but with a phase difference of $\pi/3$. The resultant emf is

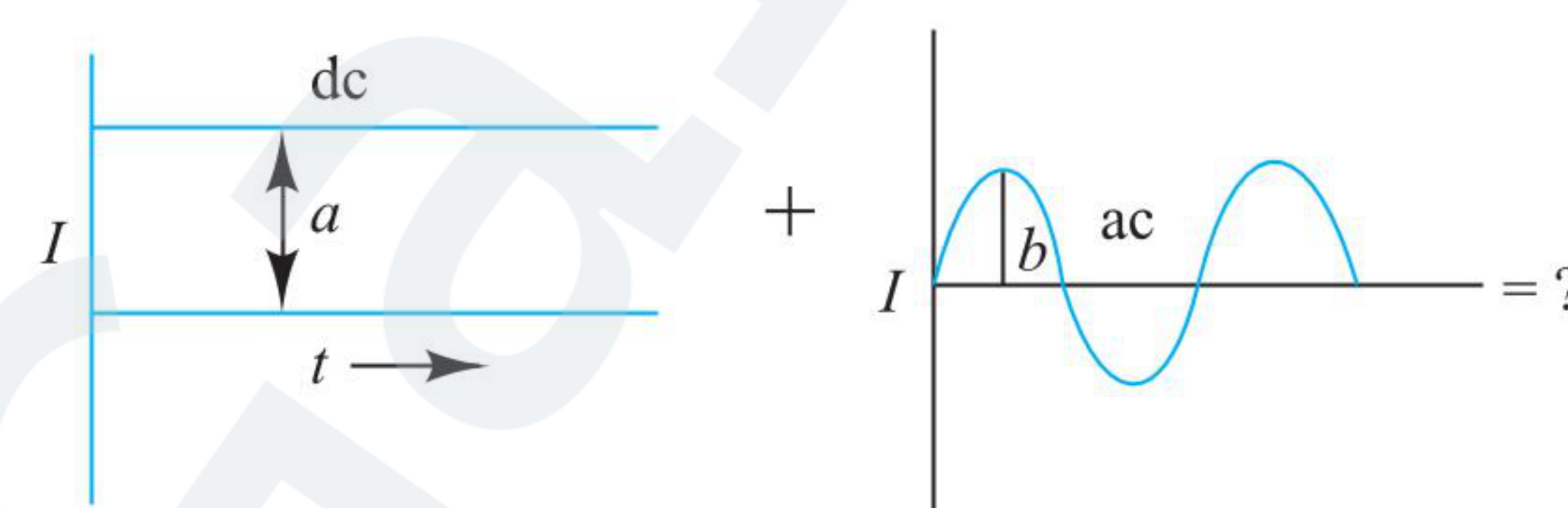
- (1) $E_0 \sin[\omega t + (\pi/3)]$ (2) $E_0 \sin[\omega t + (\pi/6)]$
(3) $\sqrt{3} E_0 \sin[\omega t + (\pi/6)]$ (4) $\sqrt{3} E_0 \sin[\omega t + (\pi/2)]$

31. For the circuit shown in figure, current in inductance is 0.8 A while that in capacitance is 0.6 A. What is the current drawn from the source?



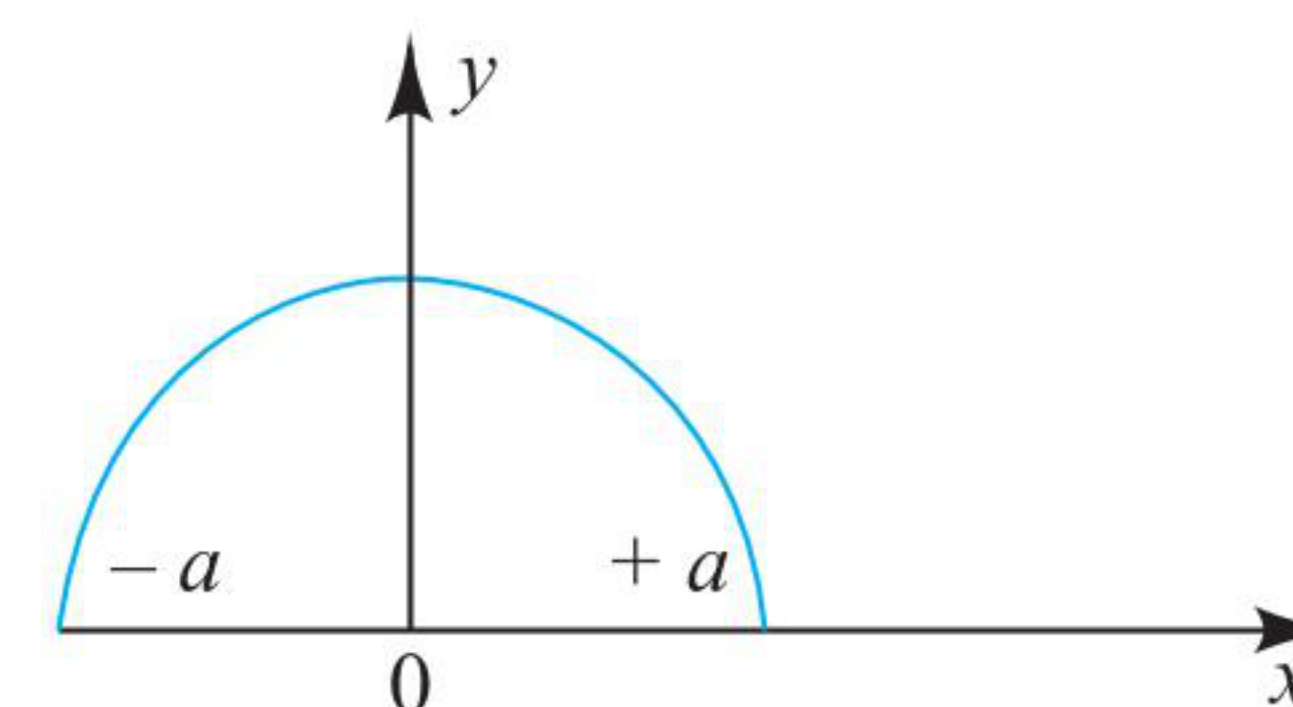
- (1) 0.1 A (2) 0.3 A
(3) 0.6 A (4) 0.2 A

32. If a direct current of value a ampere is superimposed on an alternative current $I = b \sin \omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit?



- (1) $\left[a^2 - \frac{1}{2} b^2 \right]^{1/2}$ (2) $[a^2 + b^2]^{1/2}$
(3) $\left[\frac{a^2}{2} + b^2 \right]^{1/2}$ (4) $\left[a^2 + \frac{1}{2} b^2 \right]^{1/2}$

33. Determine the rms value of a semi-circular current wave which has a maximum value of a .

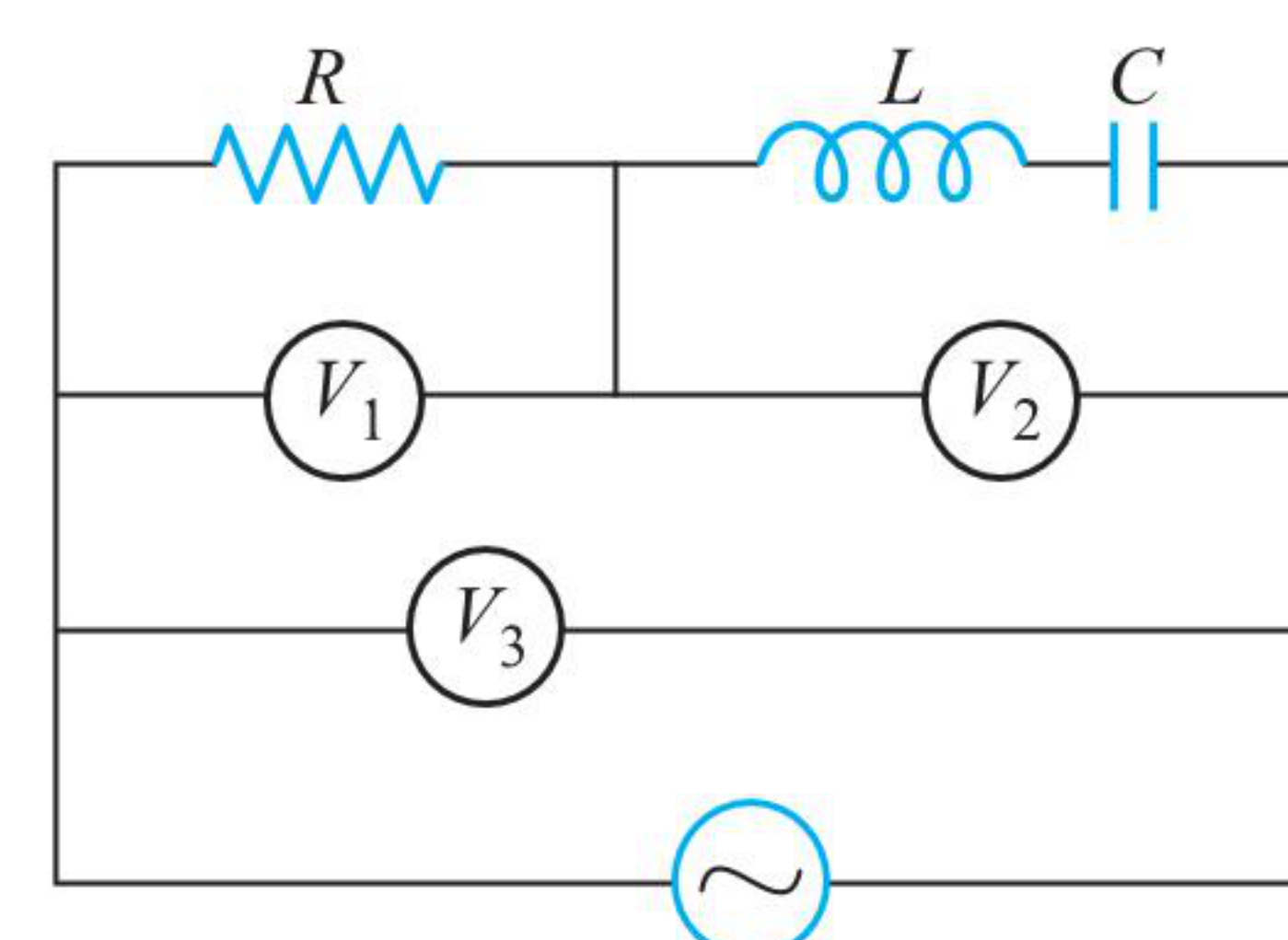


- (1) $(1/\sqrt{2}) a$ (2) $\sqrt{(3/2)} a$
(3) $\sqrt{(2/3)} a$ (4) $(1/\sqrt{3}) a$

34. An alternating voltage $E = 50\sqrt{2} \sin(100 t)$ V is connected to a 1 μF capacitor through an ac ammeter. What will be the reading of the ammeter?

- (1) 10 mA (2) 5 mA
(3) $5\sqrt{2}$ mA (4) $10\sqrt{2}$ mA

35. Which voltmeter will give zero reading at resonance?



- (1) V_1 (2) V_2
(3) V_3 (4) None

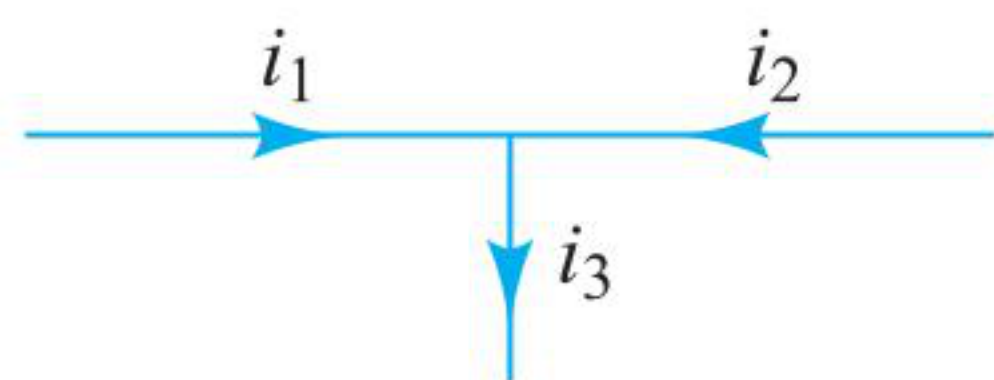
36. A 50 W, 100 V lamp is to be connected to an ac mains of 200 V, 50 Hz. What capacitor is essential to be put in series with the lamp?

- (1) $\frac{25}{\sqrt{2}} \mu\text{F}$ (2) $\frac{50}{\pi\sqrt{3}} \mu\text{F}$
 (3) $\frac{50}{\sqrt{2}} \mu\text{F}$ (4) $\frac{100}{\pi\sqrt{3}} \mu\text{F}$

37. A capacitor of $10 \mu\text{F}$ and an inductor of 1 H are joined in series. An ac of 50 Hz is applied to this combination. What is the impedance of the combination?

- (1) $\frac{5(\pi^2 - 5)}{\pi} \Omega$ (2) $\frac{10(10 - \pi^2)}{\pi} \Omega$
 (3) $\frac{10(\pi^2 - 5)}{\pi} \Omega$ (4) $\frac{5(10 - \pi^2)}{\pi} \Omega$

38. If $i_1 = 3 \sin \omega t$ and $i_2 = 4 \cos \omega t$, then i_3 is



- (1) $5 \sin(\omega t + 53^\circ)$ (2) $5 \sin(\omega t + 37^\circ)$
 (3) $5 \sin(\omega t + 45^\circ)$ (4) $5 \sin(\omega t + 53^\circ)$

39. In an ac circuit, the potential differences across an inductance and resistance joined in series are, respectively, 16 V and 20 V . The total potential difference across the circuit is

- (1) 20 V (2) 25.6 V
 (3) 31.9 V (4) 53.5 V

40. Current in an ac circuit is given by $I = 3 \sin \omega t + 4 \cos \omega t$, then

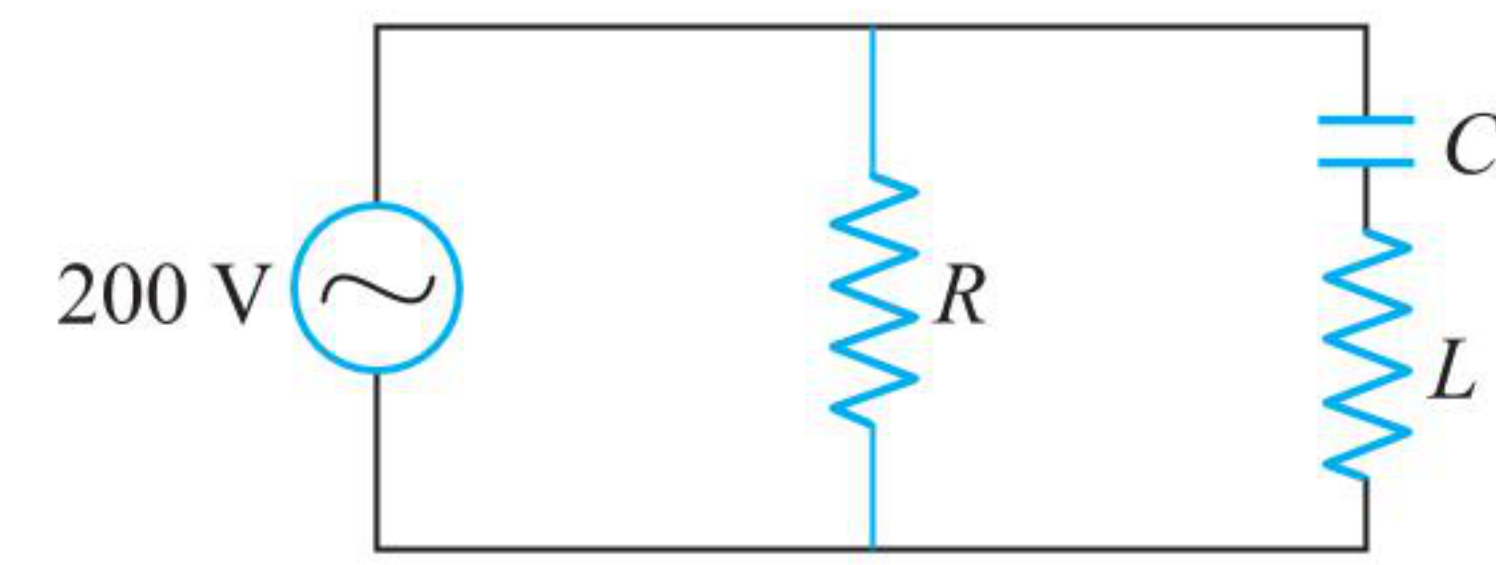
- (1) rms value of current is 5 A .
 (2) mean value of this current in any one half period will be $6/\pi$.
 (3) if voltage applied is $V = V_m \sin \omega t$, then the circuit may be containing resistance and capacitance only.
 (4) if voltage applied is $V = V_m \sin \omega t$, the circuit may contain resistance and inductance only.

41. A current source sends a current $I = i_0 \cos(\omega t)$. When connected across an unknown load, it gives a voltage output of $v = v_0 \sin[\omega t + (\pi/4)]$ across that load. Then the voltage across the current source may be brought in phase with the current through it by



- (1) connecting an inductor in series with the load
 (2) connecting a capacitor in series with the load
 (3) connecting an inductor in parallel with the load
 (4) connecting a capacitor in parallel with the load

42. In the circuit shown in figure, $X_C = 100 \Omega$, $X_L = 200 \Omega$ and $R = 100 \Omega$. The effective current through the source is



- (1) 2 A (2) $2\sqrt{2} \text{ A}$
 (3) 0.5 A (4) $\sqrt{0.4} \text{ A}$

43. For an LCR series circuit with an ac source of angular frequency ω ,

- (1) circuit will be capacitive if $\omega > \frac{1}{\sqrt{LC}}$
 (2) circuit will be inductive if $\omega = \frac{1}{\sqrt{LC}}$
 (3) power factor of circuit will be unity if capacitive reactance equals inductive reactance
 (4) current will be leading voltage if $\omega > \frac{1}{\sqrt{LC}}$

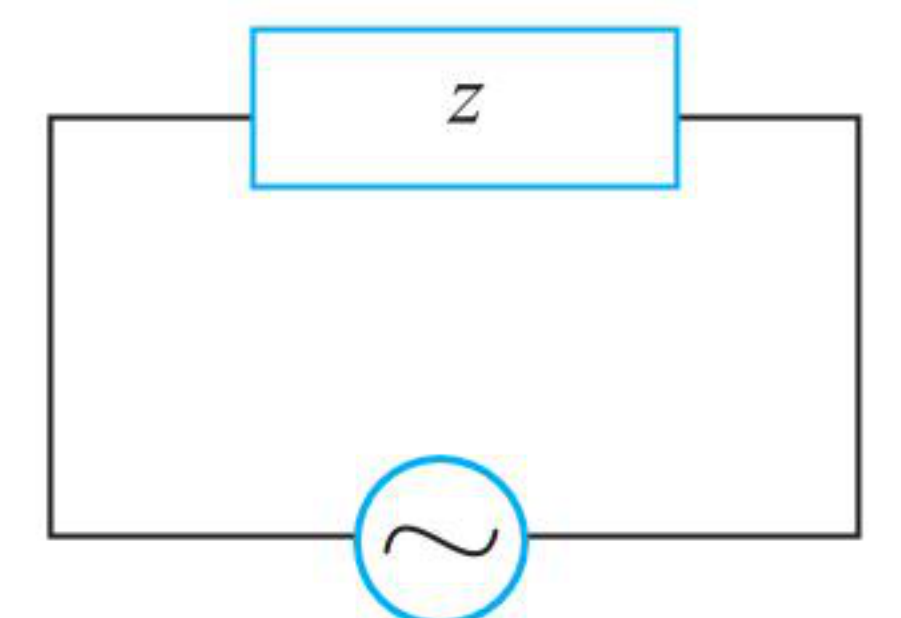
44. The value of current in two series LCR circuits at resonance is same. Then

- (1) both circuits must be having same value of capacitance and inductance
 (2) in both circuits ratio of L and C will be same
 (3) for both the circuits X_L/X_C must be same at that frequency
 (4) both circuits must have same impedance at all frequencies

45. In series LCR circuit voltage drop across resistance is 8 V , across inductor is 6 V and across capacitor is 12 V . Then

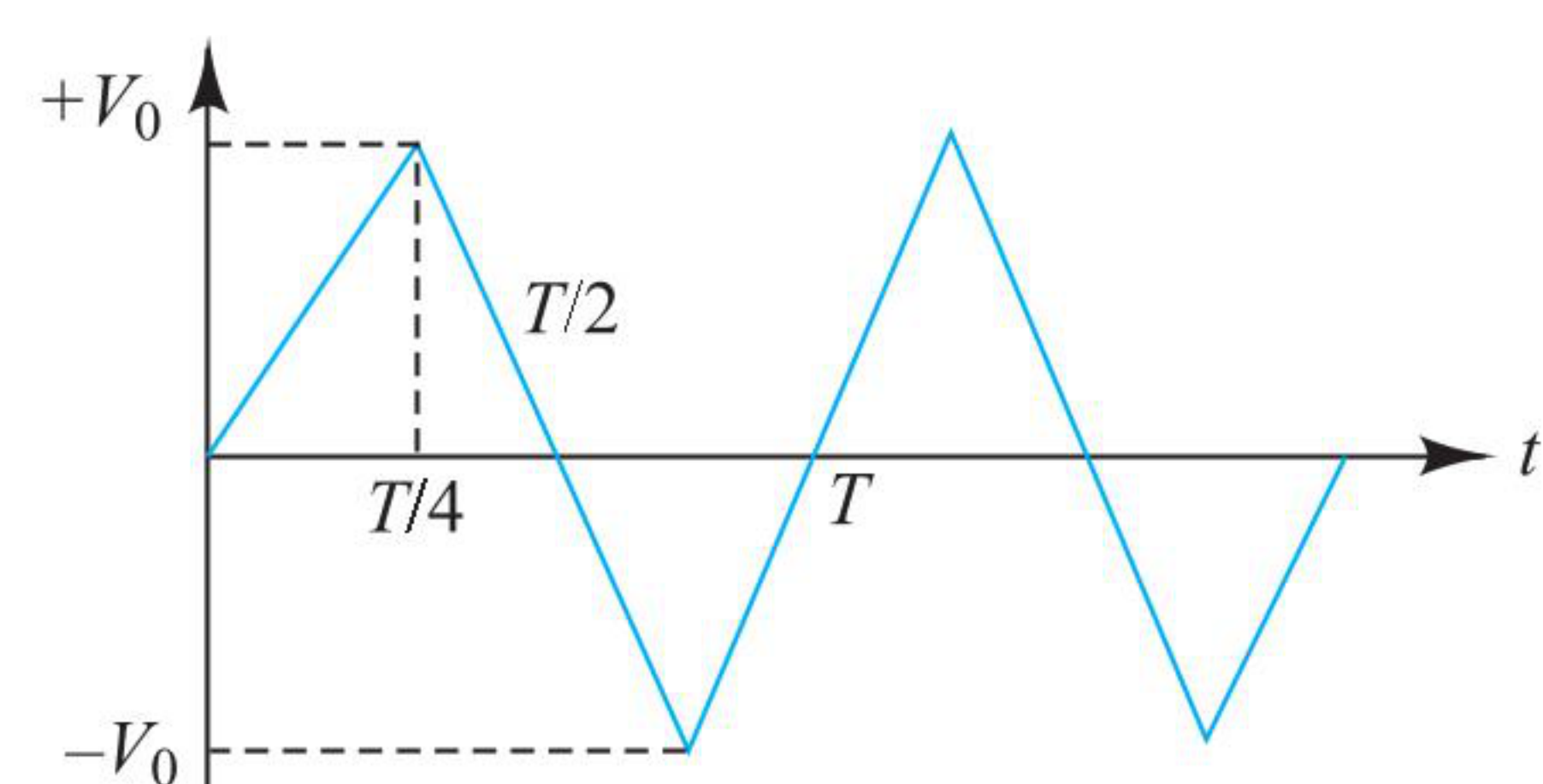
- (1) voltage of the source will be leading current in the circuit
 (2) voltage drop across each element will be less than the applied voltage
 (3) power factor of circuit will be $4/3$
 (4) none of these

46. In a black box of unknown elements (L or R or any other combination), an ac voltage $E = E_0 \sin(\omega t + \phi)$ is applied and current in the circuit was found to be $I = I_0 \sin[\omega t + \phi + (\pi/4)]$. Then the unknown elements in the box may be



- (1) only capacitor
 (2) inductor and resistor both
 (3) either capacitor, resistor, and inductor or only capacitor and resistor
 (4) only resistor

47. The voltage time ($V-t$) graph for triangular wave having peak value V_0 is as shown in figure.



The rms value of V in time interval from $t = 0$ to $T/4$ is

- (1) $\frac{V_0}{\sqrt{3}}$ (2) $\frac{V_0}{2}$
 (3) $\frac{V_0}{\sqrt{2}}$ (4) None of these

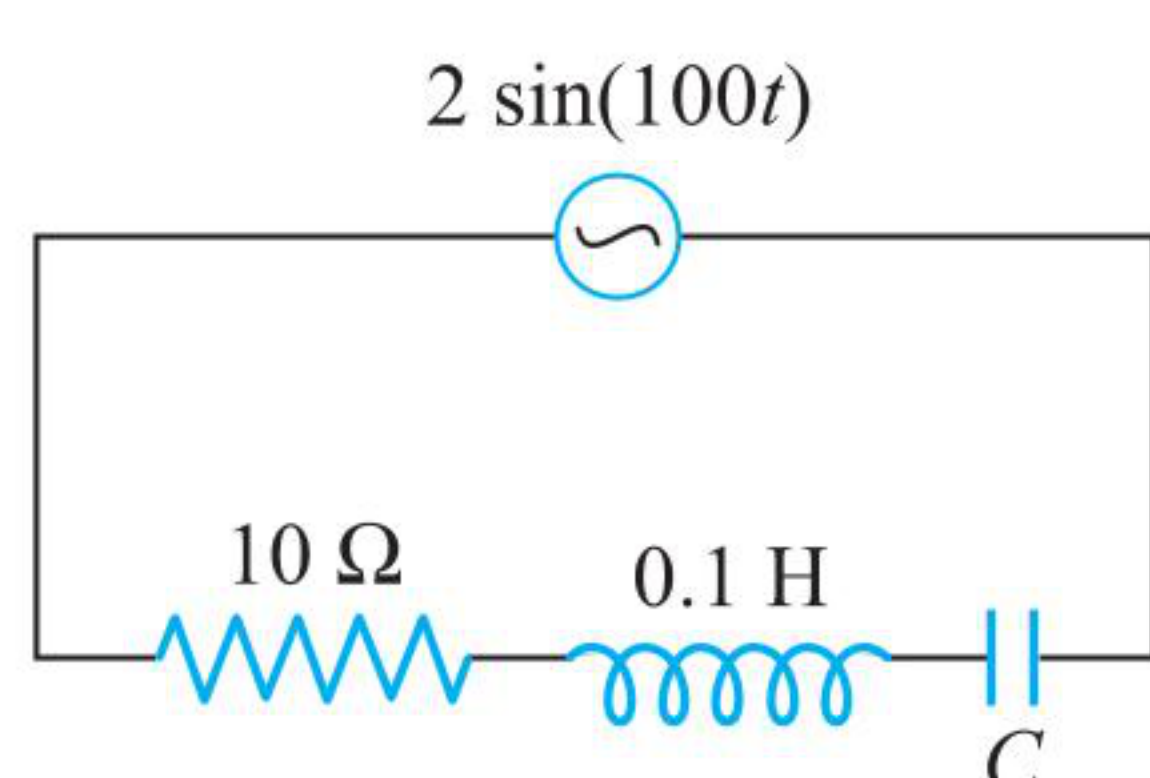
48. In the above question, the average value of voltage (V) in one time period will be

- (1) $\frac{V_0}{\sqrt{3}}$ (2) $\frac{V_0}{2}$
 (3) $\frac{V_0}{\sqrt{2}}$ (4) 0

49. What reading would you expect of a square-wave current, switching rapidly between $+0.5$ A and -0.5 A, when passed through an ac ammeter?

- (1) 0 (2) 0.5 A
 (3) 0.25 A (4) 1.0 A

50. The power factor of the circuit in figure is $1/\sqrt{2}$. The capacitance of the circuit is equal to



- (1) 400 μ F (2) 300 μ F
 (3) 500 μ F (4) 200 μ F

51. In an ideal transformer, the voltage and the current in the primary coil are 200 V and 2 A, respectively. If the voltage in the secondary coil is 2000 V, then the value of current in the secondary coil will be

- (1) 0.2 A (2) 2 A
 (3) 10 A (4) 20 A

52. An AC source rated 100 V (rms) supplies a current of 10 A (rms) to a circuit. The average power delivered by the source:

- (1) must be 1000 W
 (2) may be greater than 1000 W
 (3) may be less than 1000 W
 (4) all of the above three are possible

53. The voltage of an AC source varies with time according to the relation: $E = 120 \sin 100 \pi t \cos 100 \pi t$ V. What is the peak voltage of the source?

- (1) 60 V (2) 120 V
 (3) 30 V (4) $\sqrt{2}$ V

54. A resistance of 20Ω is connected to a source of an alternating potential $V = 220 \sin(100 \pi t)$. The time taken by the current to change from the peak value to rms value is

- (1) 0.2 s (2) 0.25 s
 (3) 25 s (4) 2.5×10^{-3} s

55. A current is made up of two components 3 A dc component and ac component given by $I = 4 \sin \omega t$ A. The effective value of current is

- (1) $\sqrt{17}$ A (2) $\sqrt{7}$ A
 (3) $\sqrt{25}$ A (4) $\sqrt{7 \times 25}$ A

56. An alternating e.m.f. of 200 V and 50 cycles is connected to a circuit of resistance 3.142Ω and inductance 0.01 H. The lag in time between the e.m.f. and the current is

- (1) 1.5 ms (2) 2.5 ms
 (3) 3.5 ms (4) None of these

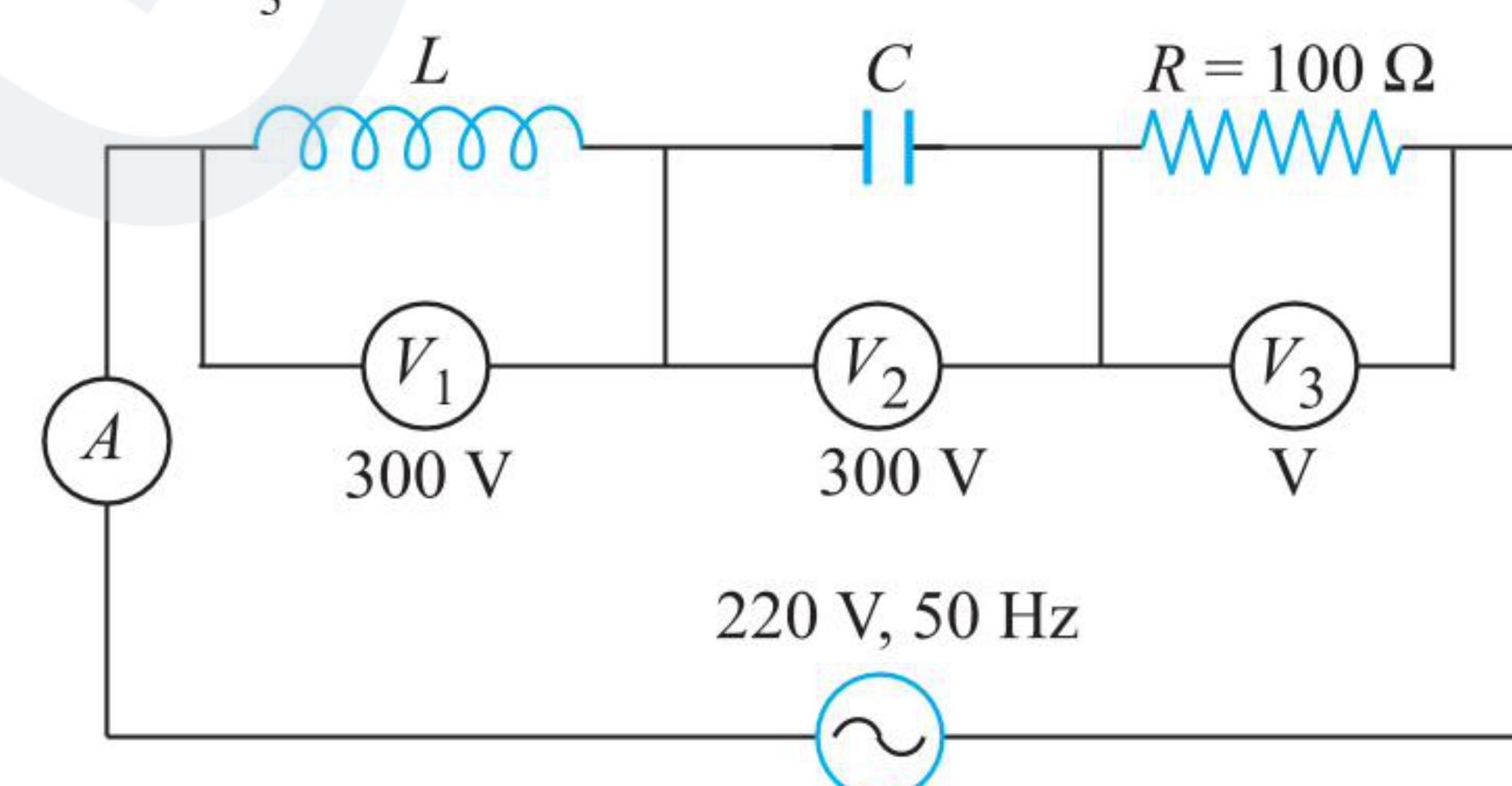
57. In an AC circuit, a resistance of R ohm is connected in series with an inductance L . If phase angle between voltage and current by 45° , the value of inductive reactance will be

- (1) $R/4$
 (2) $R/2$
 (3) R
 (4) cannot be found with the given data

58. A direct current of 2 A and an alternating current having a maximum value of 2 A flow through two identical resistances. The ratio of heat produced in the two resistances will be

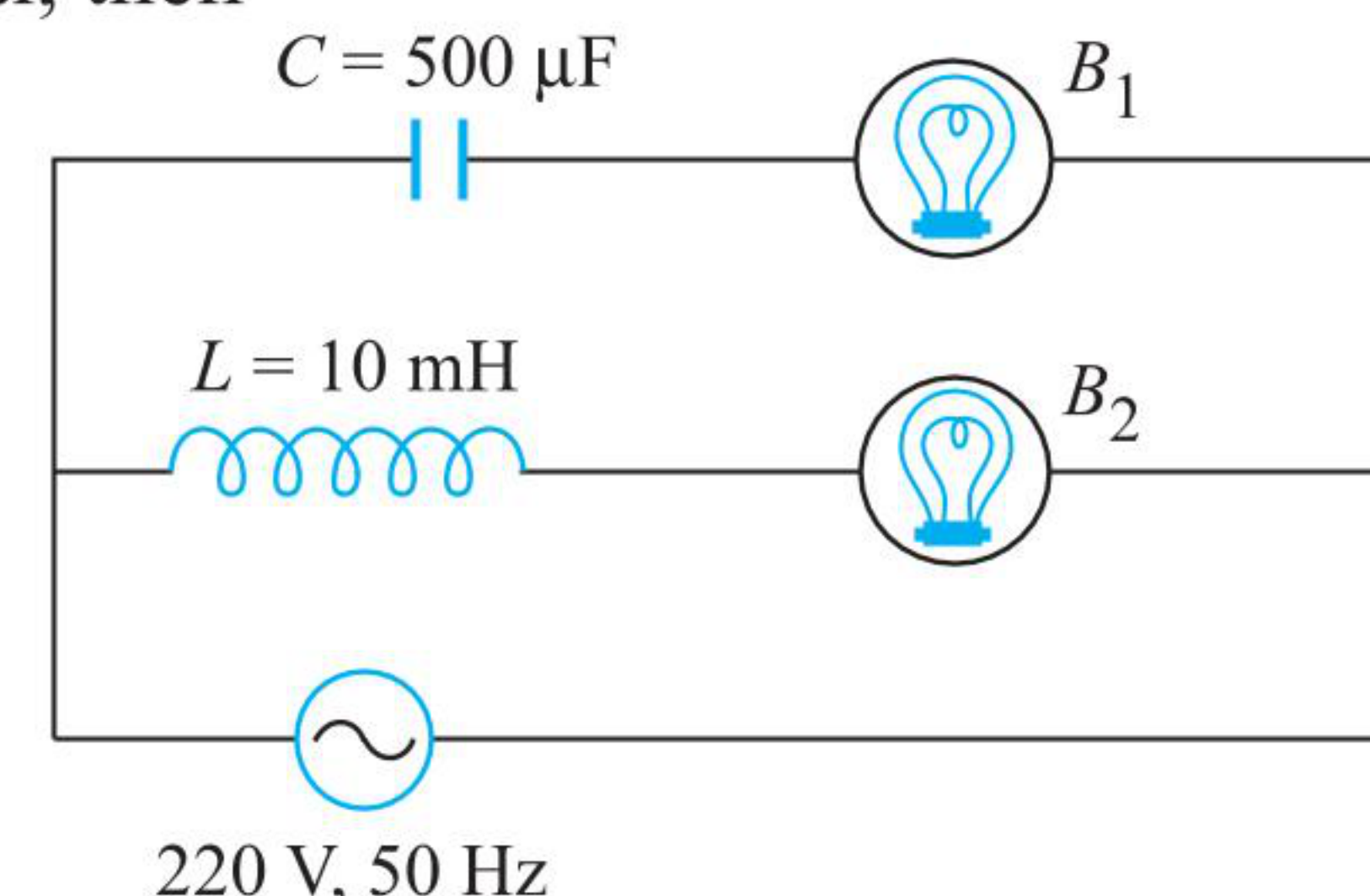
- (1) 1 : 1 (2) 1 : 2
 (3) 2 : 1 (4) 4 : 1

59. In the circuit shown in figure, what will be the reading of the voltmeter V_3 and ammeter A ?



- (1) 800 V, 2 A (2) 300 V, 2 A
 (3) 220 V, 2.2 V (4) 100 V, 2 A.

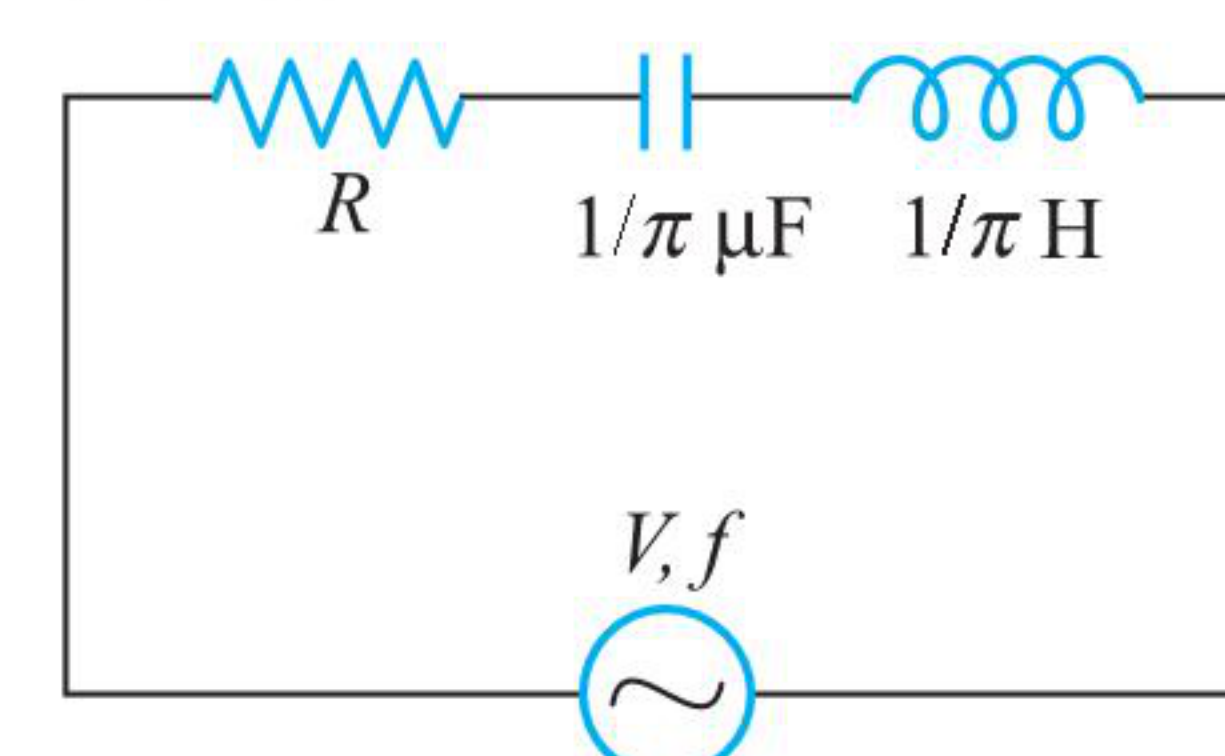
60. In the circuit shown in figure, if both the bulbs B_1 and B_2 are identical, then



- (1) their brightness will be the same
 (2) B_2 will be brighter than B_1
 (3) as frequency and that of B_2 will decrease
 (4) only B_2 will glow because the capacitor has infinite impedance

Multiple Correct Answers Type

1. In an ac circuit shown below in figure, the supply voltage has a constant rms value V but variable frequency f . At resonance, the circuit



- (1) has current I given by $I = \frac{V}{R}$
 (2) has a resonance frequency 500 Hz
 (3) has a voltage across the capacitor which is 180° out of phase with that across the inductor

(4) has a current given by $I = \frac{V}{\sqrt{R^2 + \left(\frac{1}{\pi} + \frac{1}{\pi}\right)^2}}$

2. Resonance occurs in a series LCR circuit when the frequency of the applied emf is 1000 Hz.

- (1) when frequency = 900 Hz, then the current through the voltage source will be ahead of emf of the source
 (2) the impedance of the circuit is minimum at $f = 1000$ Hz
 (3) at only resonance the voltages across L and C differ in phase by 180°
 (4) if the value of C is doubled, resonance occurs at $f = 2000$ Hz

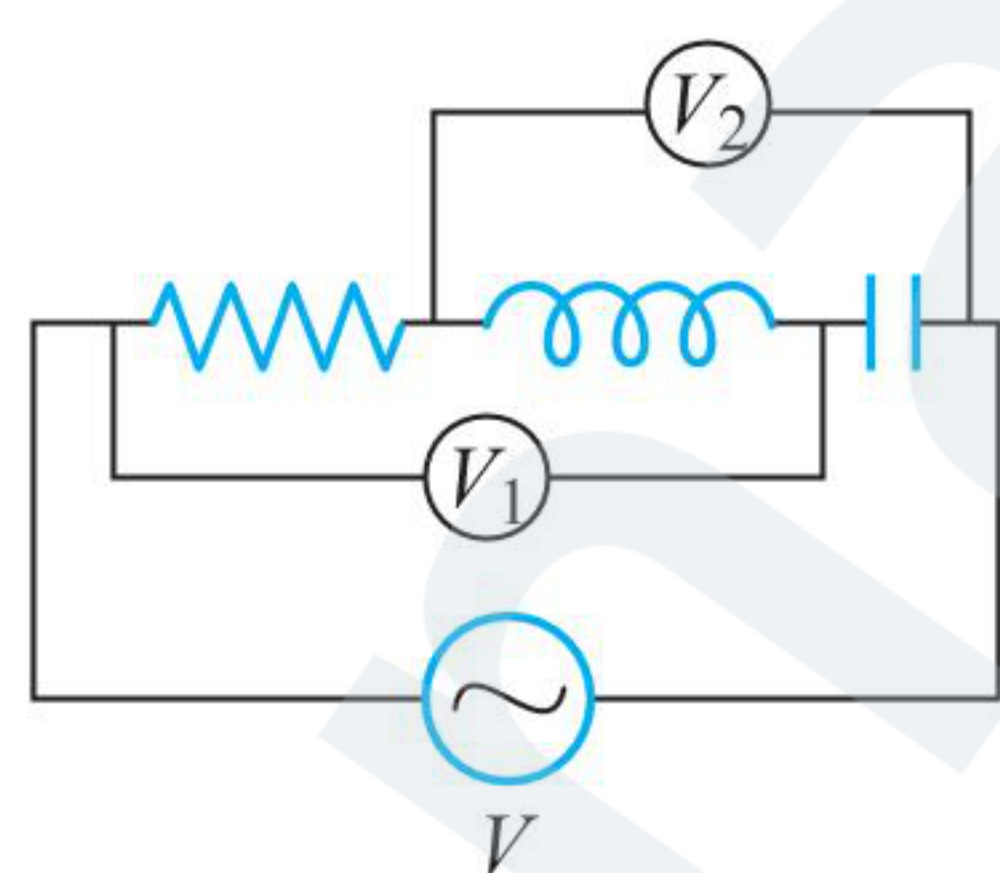
3. Which of the following statements is true? Heat produced in a current carrying conductor depends upon

- (1) the time for which the current flows in the conductor
 (2) the resistance of the conductor
 (3) the strength of the current
 (4) the nature of current (ac or dc)

4. A choke coil of resistance 5Ω and inductance 0.6 H is in series with a capacitance of $10 \mu\text{F}$. If a voltage of 200 V is applied and the frequency is adjusted to resonance, the current and voltage across the inductance and capacitance are I_0 , V_0 , and V_1 , respectively. We have

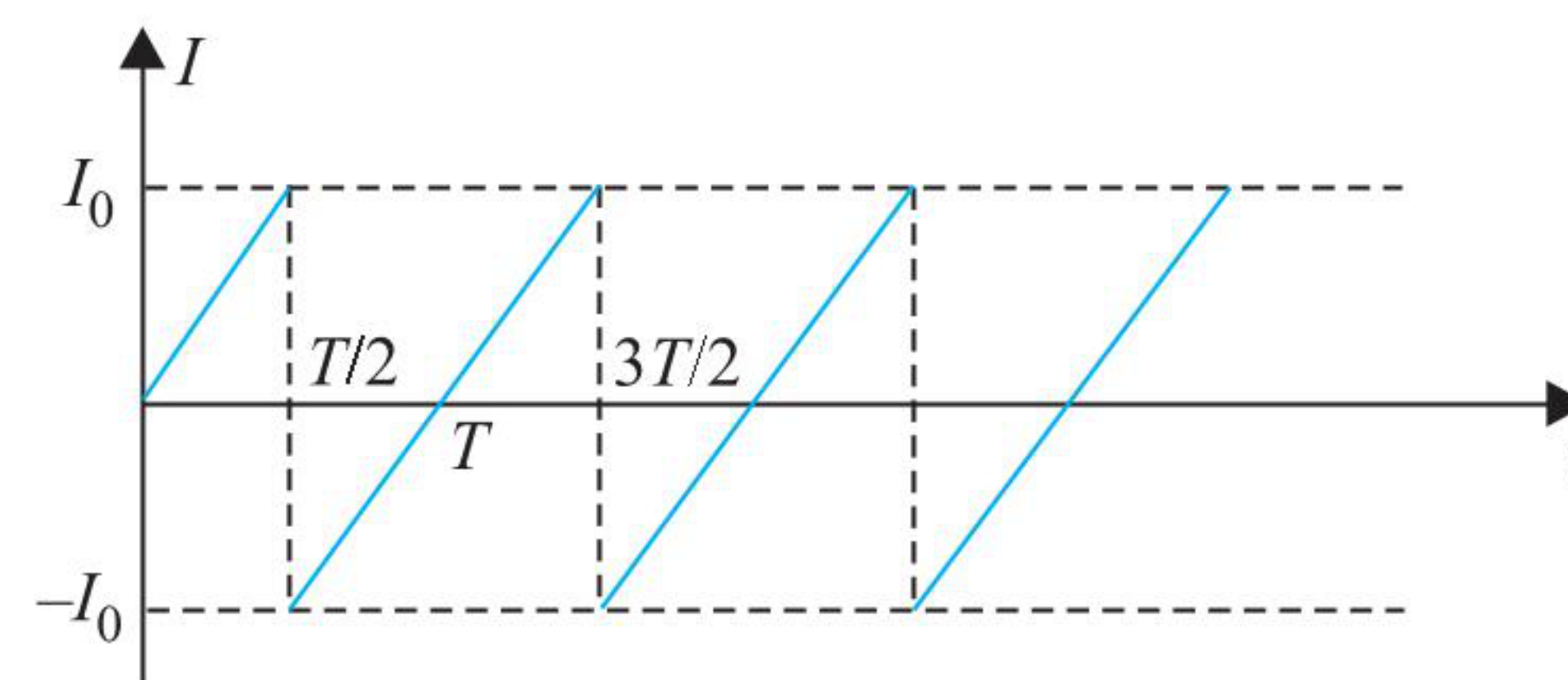
- (1) $I_0 = 40$ A (2) $V_0 = 9.8$ kV
 (3) $V_1 = 9.8$ kV (4) $V_1 = 19.6$ kV

5. In an RLC series circuit shown in figure, the readings of voltmeters V_1 and V_2 are 100 V and 120 V, respectively. The source voltage is 130 V. For this situation, mark out the correct statement(s).



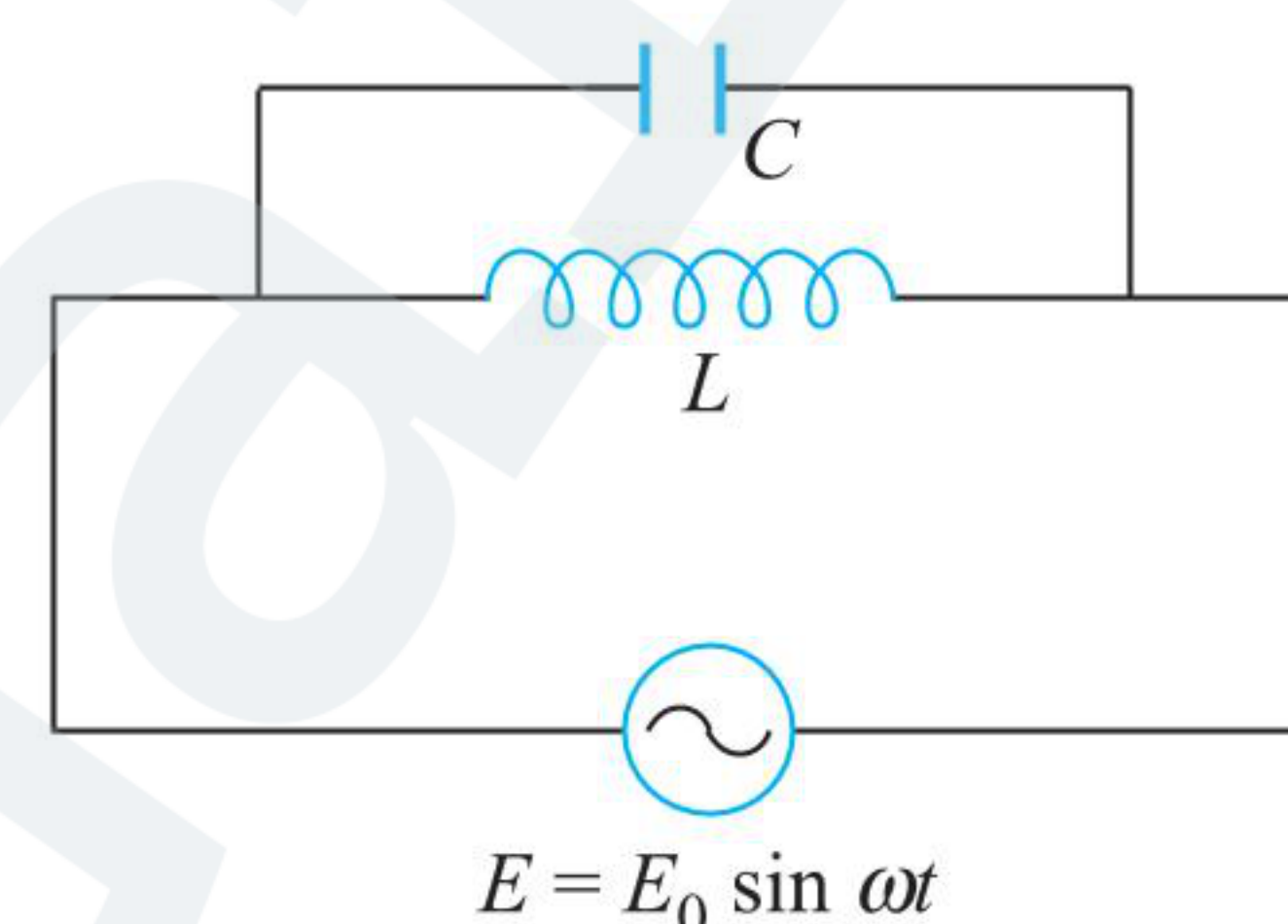
- (1) Voltage across resistor, inductor and capacitor are 50 V, $50\sqrt{3}$ V, and $120 + 50\sqrt{3}$ V, respectively
 (2) Voltage across resistor, inductor, and capacitor are 50 V, $50\sqrt{3}$ V, and $120 - 50\sqrt{3}$ V, respectively
 (3) Power factor of the circuit is $\frac{5}{13}$
 (4) The circuit is capacitive in nature

6. The current in a certain circuit varies with time as shown in figure. All the straight lines are parallel to each other. Then for time interval $T/2$ to $3T/2$



- (1) average current is zero (2) average current is $I_0/\sqrt{3}$
 (3) rms current is $I_0/\sqrt{3}$ (4) rms current is $2I_0/\sqrt{3}$

7. For the circuit shown in figure, the emf of the generator is E . The current through the inductor is 1.6 A, while the current through the condenser is 0.4 A. Then

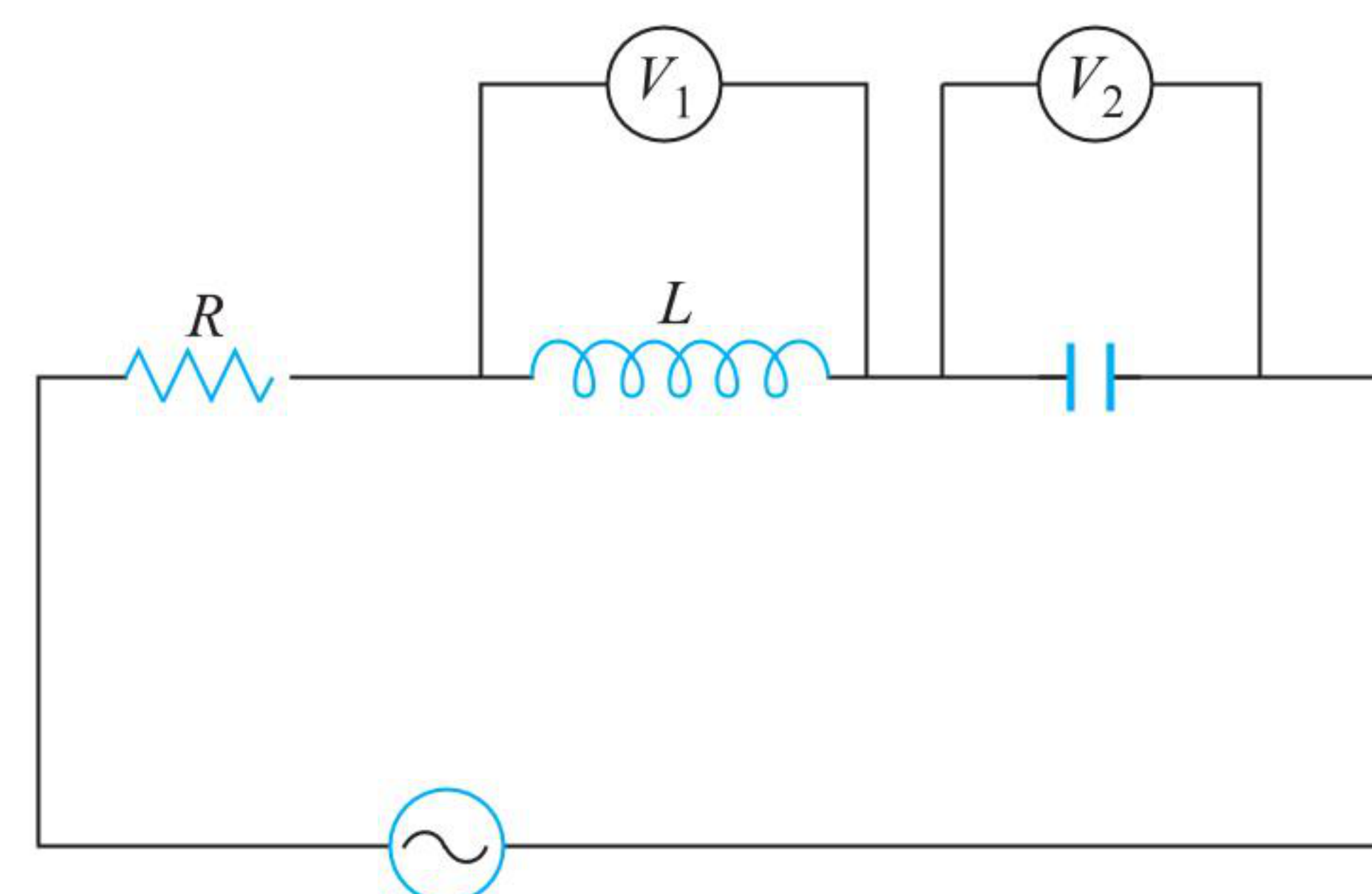


- (1) current drawn from the generator is $I = 2$ A
 (2) current drawn from the generator is $I = 1.2$ A

(3) $\omega = \frac{1}{2\sqrt{LC}}$

(4) $\omega = \frac{1}{4\sqrt{LC}}$

8. In figure shown $R = 100 \Omega$, $L = \frac{2}{\pi}$ H and $C = \frac{8}{\pi} \mu\text{F}$ are connected in series with an ac source of 200 V and frequency f . V_1 and V_2 are two hot-wire voltmeters. If the readings of V_1 and V_2 are same, then



- (1) $f = 125$ Hz
 (2) $f = 250$ p Hz
 (3) Current through R is 2 A
 (4) $V_1 = V_2 = 1000$ V

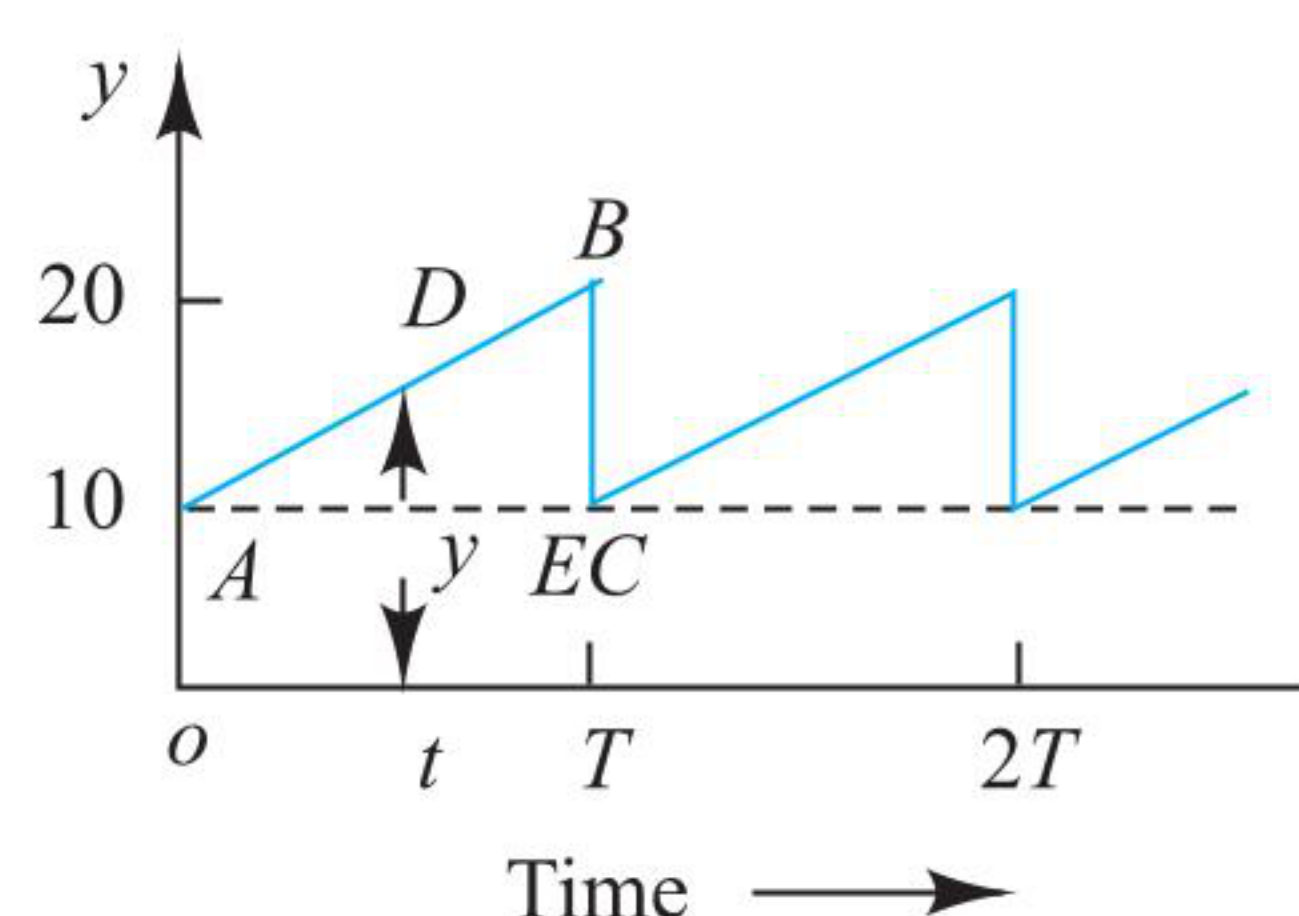
Linked Comprehension Type

For Problems 1–3

If the voltage in an ac circuit is represented by the equation, $V = 220\sqrt{2} \sin(314t - \phi)$, calculate

1. rms value of the voltage

- (1) 220 V (2) 314 V
 (3) $220\sqrt{2}$ V (4) $200/\sqrt{2}$ V
2. average voltage
 (1) 220 V (2) $622/\pi$ V
 (3) $220/\sqrt{2}$ V (4) $220\sqrt{2}$ V
3. frequency of ac
 (1) 50 Hz (2) $50\sqrt{2}$ Hz
 (3) $50/\sqrt{2}$ Hz (4) 75 Hz

For Problems 4–5


4. The average value of the wave-form shown in figure is.
 (1) $15\sqrt{2}$ (2) $10\sqrt{2}$
 (3) 10 (4) 15
5. The rms value of the signal is
 (1) $10\sqrt{\frac{7}{3}}$ (2) $\frac{10}{\sqrt{3}}$
 (3) $10\sqrt{7}$ (4) $10\sqrt{3}$

For Problems 6–7

A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12 V, 50 rad s⁻¹ ac source, a current of 2.4 A flows in the circuit.

6. The inductance of the coil is
 (1) 0.02 H (2) 0.04 H
 (3) 0.08 H (4) 1.0 H
7. The power developed in the circuit of a 2500 μ F capacitor that is connected in series with the coil is
 (1) 17.28 W (2) 8.64 W
 (3) 10 W (4) 15 W

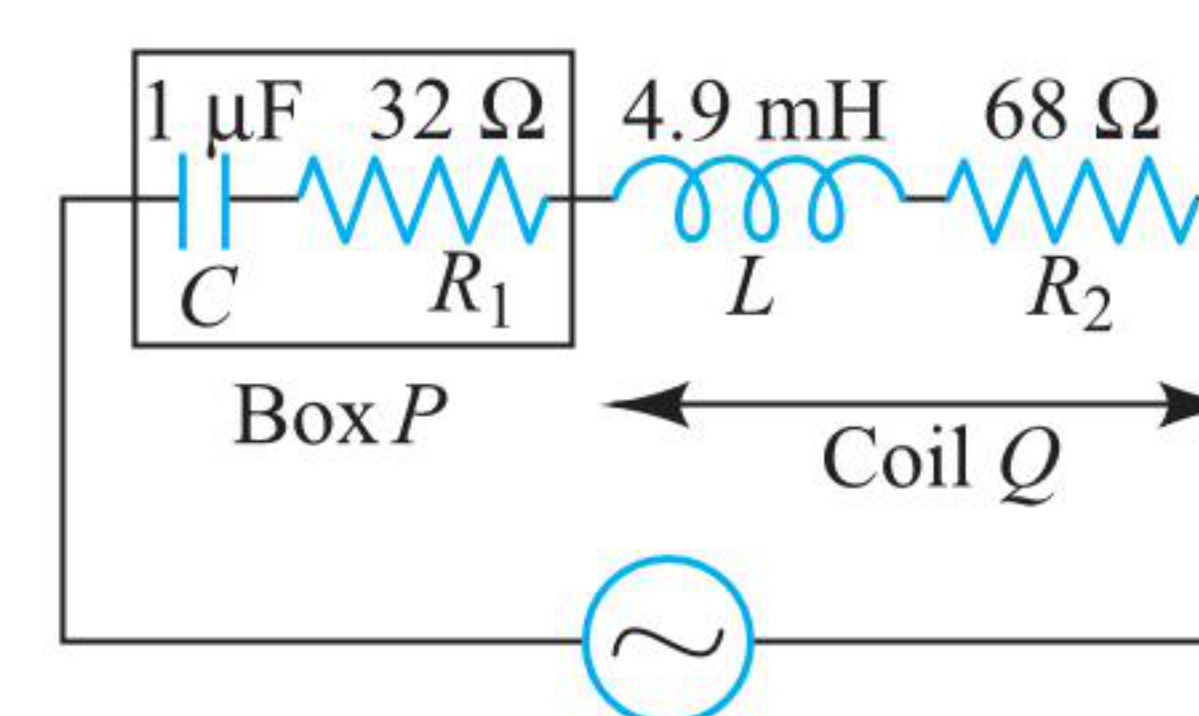
For Problems 8–9

An inductor 20×10^{-3} H, a capacitor 100 μ F, and a resistor 50 Ω are connected in series across a source of emf $V = 10 \sin 314 t$.

8. Then the energy dissipated in the circuit in 20 min is
 (1) 960 J (2) 900 J
 (3) 250 J (4) 500 J
9. If resistance is removed from the circuit and the value of inductance is doubled, the variation of current with time in the new circuit is
 (1) $0.52 \cos 314 t$ (2) $0.52 \sin 314 t$
 (3) $0.52 \sin(314 t + \pi/3)$ (4) None of these

For Problems 10–13

A box *P* and a coil *Q* are connected in series with an ac source of variable frequency. The emf of the source is constant at 10 V. Box *P* contains a capacitance of 1 μ F in series with a resistance of 32 Ω . Coil *Q* has a self inductance of 4.9 mH and a resistance of 68 Ω in series. The frequency is adjusted so that maximum current flows in *P* and *Q*.



10. The impedance of *P* at this frequency is
 (1) 77 Ω (2) 36 Ω
 (3) 40 Ω (4) 125 Ω
11. The impedance of *Q* at this frequency is
 (1) 200 Ω (2) $\sqrt{1350}$ Ω
 (3) 55 Ω (4) $\sqrt{9524}$ Ω
12. The voltage across *P* is
 (1) 12 V (2) 7.7 V
 (3) 10 V (4) 24 V
13. The voltage across *Q* is
 (1) 20 V (2) $\frac{\sqrt{1350}}{10}$ V
 (3) 5.5 V (4) $\frac{\sqrt{9524}}{10}$ V

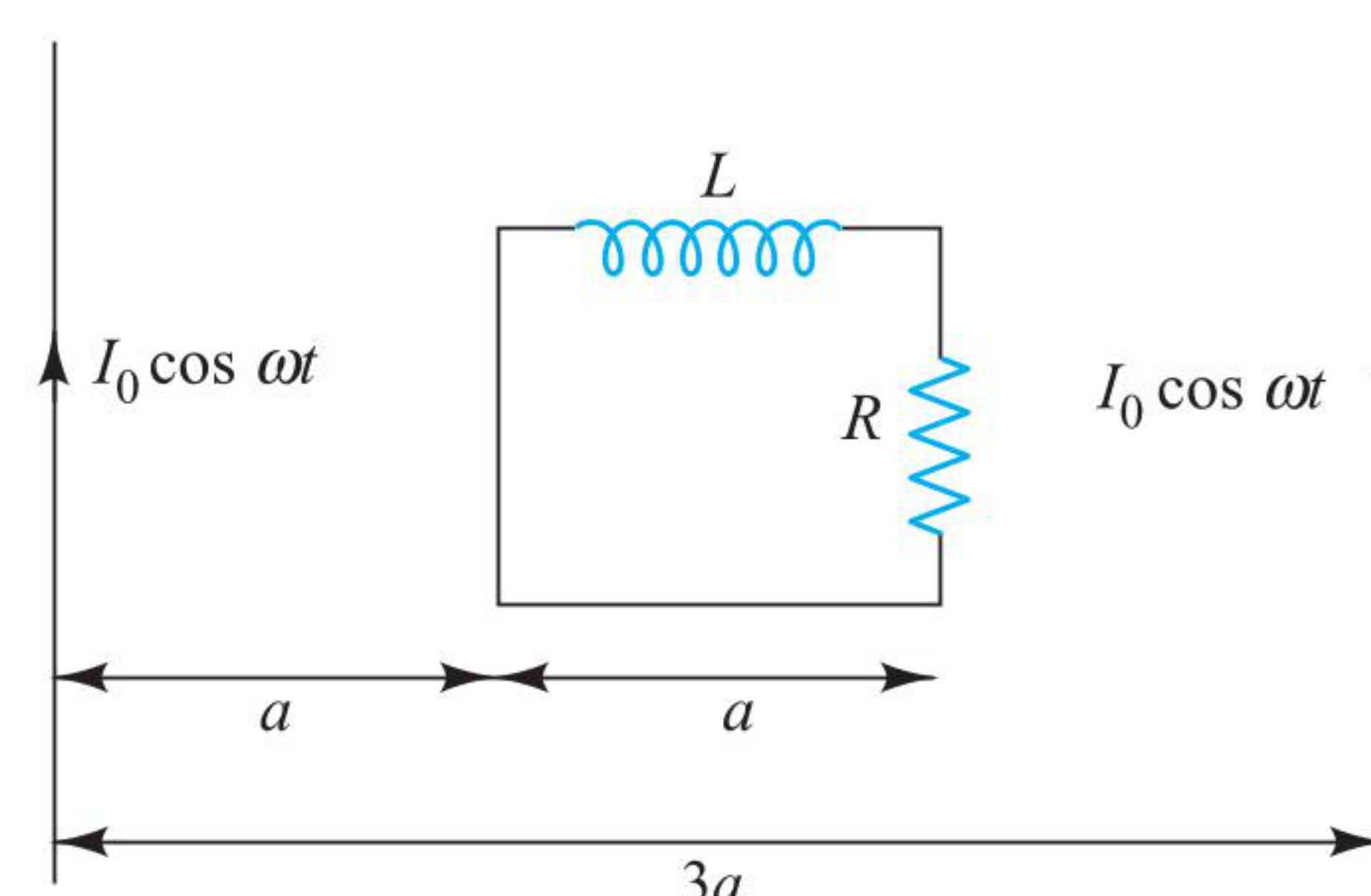
For Problems 14–16

A series LCR circuit containing a resistance of 120 Ω has angular resonance frequency 4×10^5 rad s⁻¹. At resonance the voltages across resistance and inductance are 60 V and 40 V, respectively.

14. The value of inductance *L* is
 (1) 0.1 mH (2) 0.2 mH
 (3) 0.35 mH (4) 0.4 mH
15. The value of capacitance *C* is
 (1) $\frac{1}{32}$ μ F (2) $\frac{1}{16}$ μ F
 (3) 32 μ F (4) 16 μ F
16. At what frequency, the current in the circuit lags the voltage by 45°?
 (1) 4×10^5 rad s⁻¹ (2) 3×10^5 rad s⁻¹
 (3) 8×10^5 rad s⁻¹ (4) 2×10^5 rad s⁻¹

For Problems 17–19

In figure, a square loop consisting of an inductor of inductance *L* and resistor of resistance *R* is placed between two long parallel wires. The two long straight wires have time-varying current of magnitude $I = I_0 \cos \omega t$ but the directions of current in them are opposite.



17. Total magnetic flux in this loop is

- (1) $\frac{\mu_0 I a}{\pi} \ln 2$ (2) $\frac{2\mu_0 I a}{\pi} \ln 2$
 (3) $\frac{4\mu_0 I a}{\pi} \ln 2$ (4) $\frac{\mu_0 I a}{2\pi} \ln 2$

18. Magnitude of emf in this circuit only due to flux change associated with two long straight current carrying wires will be

- (1) $\frac{\mu_0 a \ln 2 I_0 \omega}{\pi} \sin \omega t$ (2) $\frac{2\mu_0 a \ln 2 I_0 \omega}{\pi} \sin \omega t$
 (3) $\frac{\mu_0 a \ln 2 I_0 \omega}{2\pi} \cos \omega t$ (4) $\frac{\mu_0 a \ln 2 I_0 \omega}{\pi} \cos \omega t$

19. The instantaneous current in the circuit will be

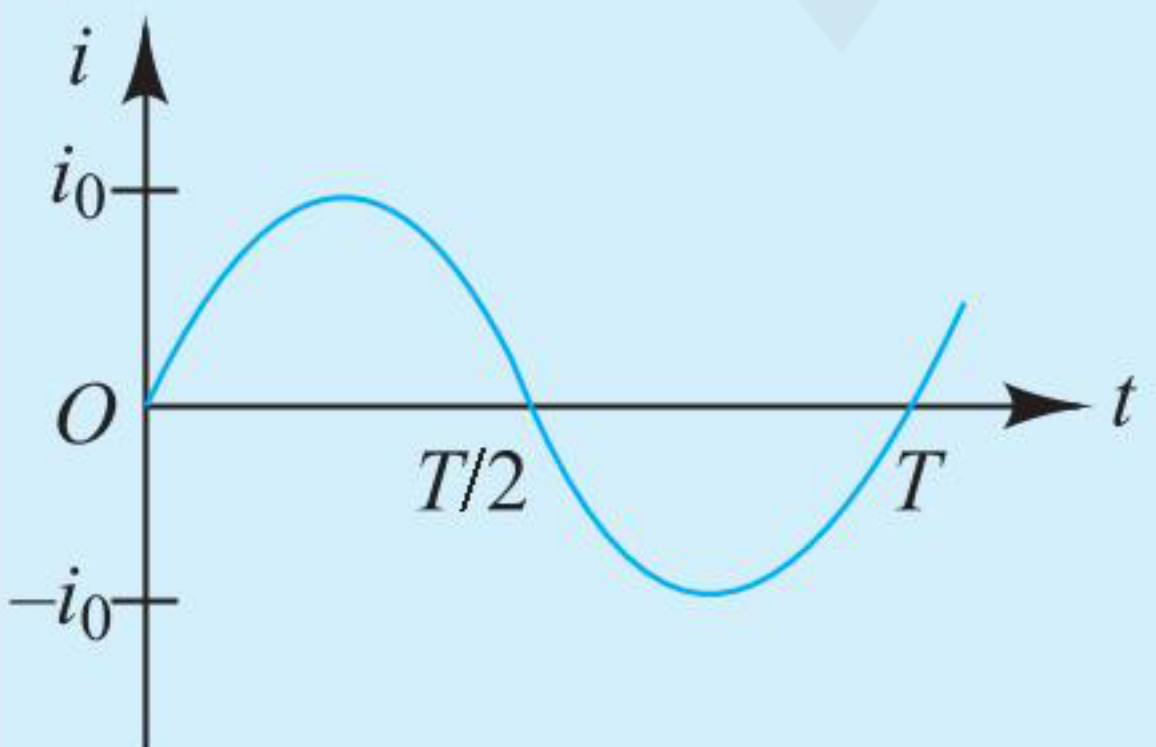
- (1) $\frac{2\mu_0 a \ln 2 I_0 \omega}{\pi \sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi)$
 (2) $\frac{2\mu_0 a \ln 2 I_0 \omega}{\pi \sqrt{R^2 + \omega^2 L^2}} \sin(\omega t + \phi)$
 (3) $\frac{\mu_0 a \ln 2 I_0 \omega}{\pi \sqrt{R^2 + \omega^2 L^2}} \sin \omega t$
 (4) $\frac{\mu_0 a \ln 2 I_0 \omega}{\pi \sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi]$
 (where $\tan \phi = \frac{\omega L}{R}$)

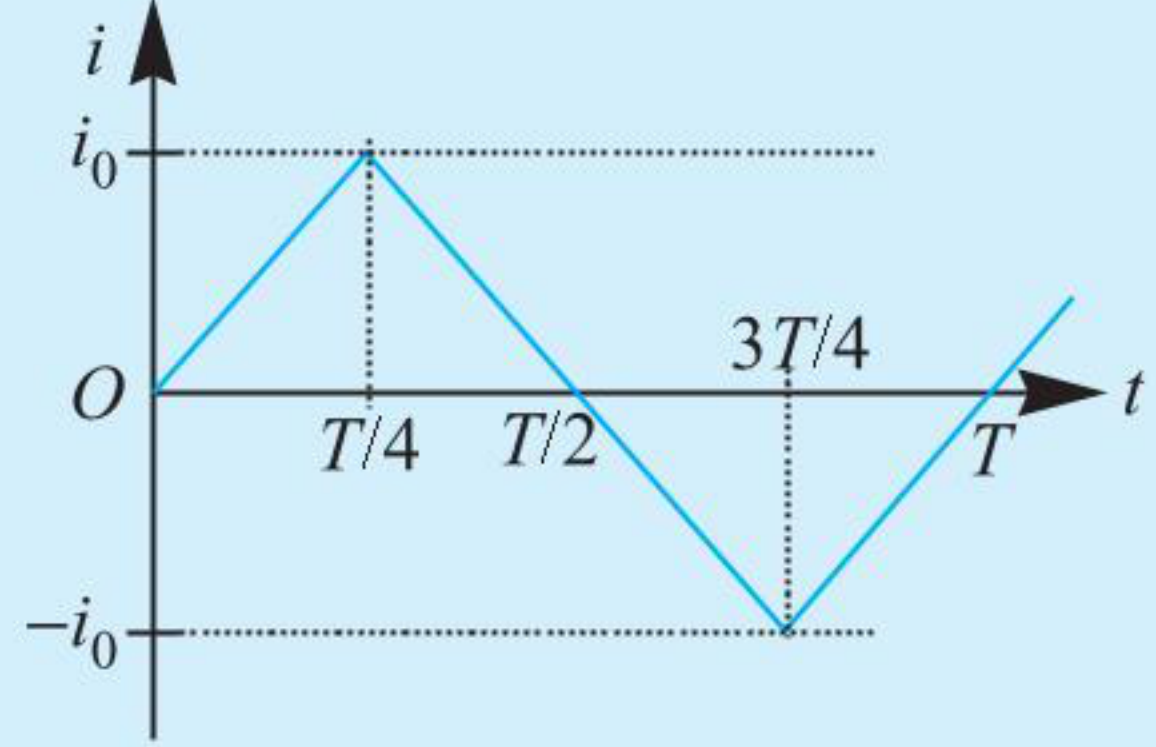
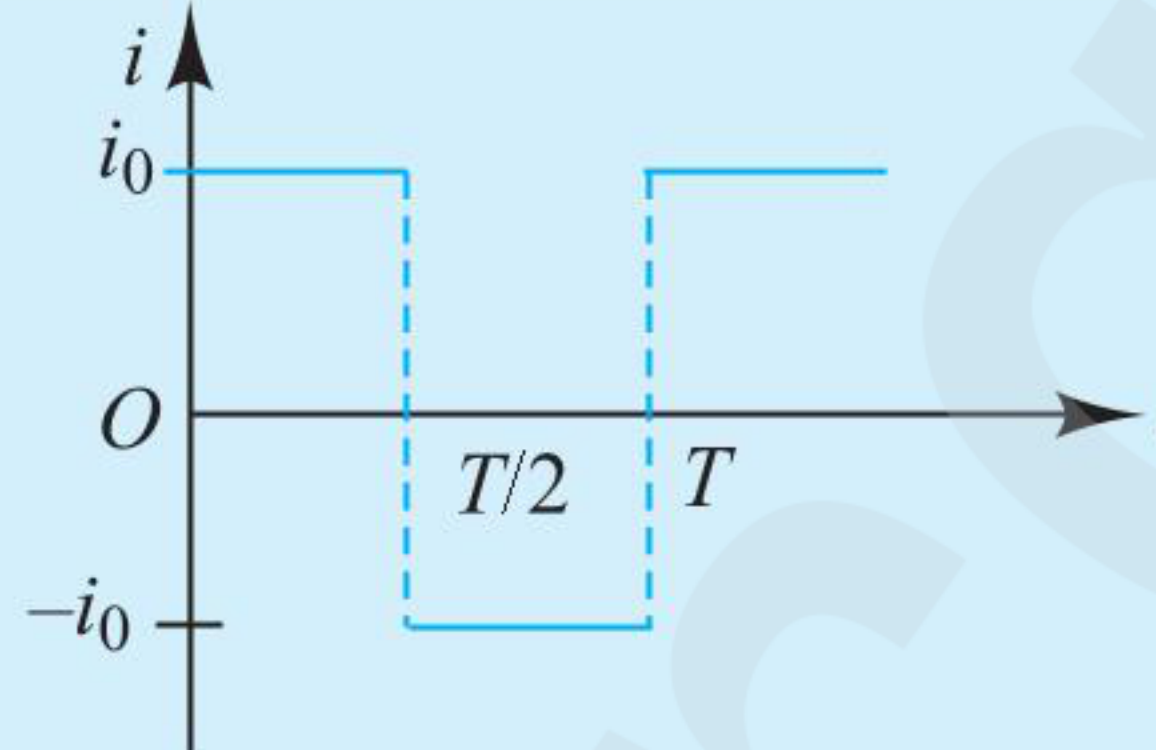
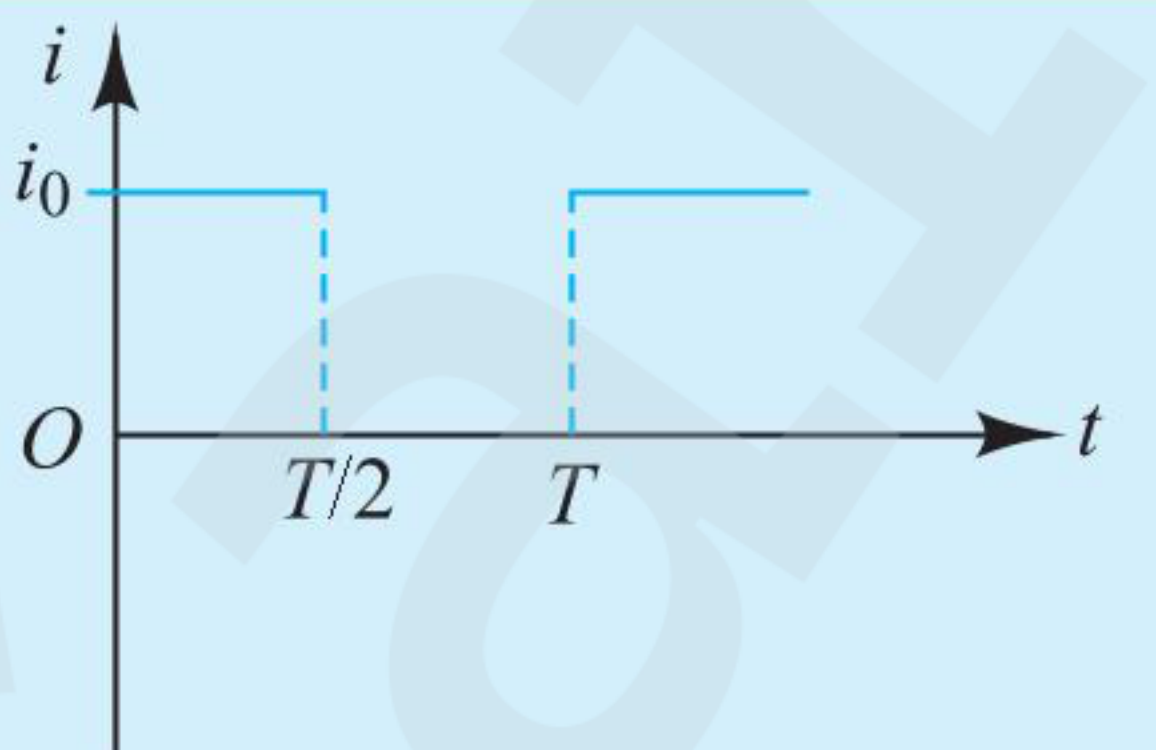
Matrix Match Type

1. Match the following column.

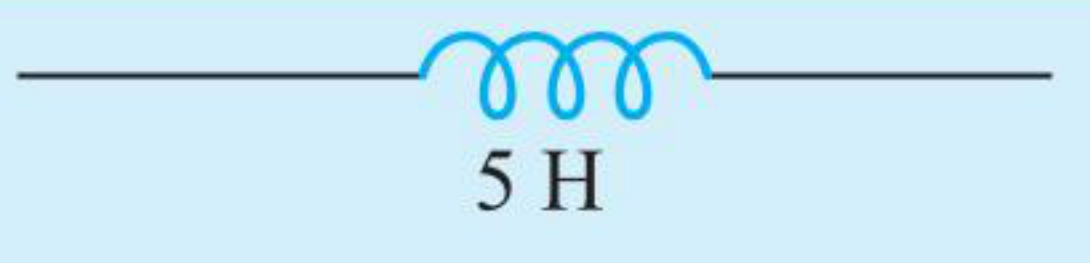
Column I		Column II	
i.	Inductance of a coil	a.	Depends on resistivity
ii.	Capacitance	b.	Depends on shape
iii.	Impedance of coil	c.	Depends on medium inserted
iv.	Reactance of a capacitor	d.	Depends on external voltage source

2. In column I, variation of current i with time t is given in the figure. In column II, root mean square current i_{rms} and average current are given. Match column I with corresponding quantities given in column II

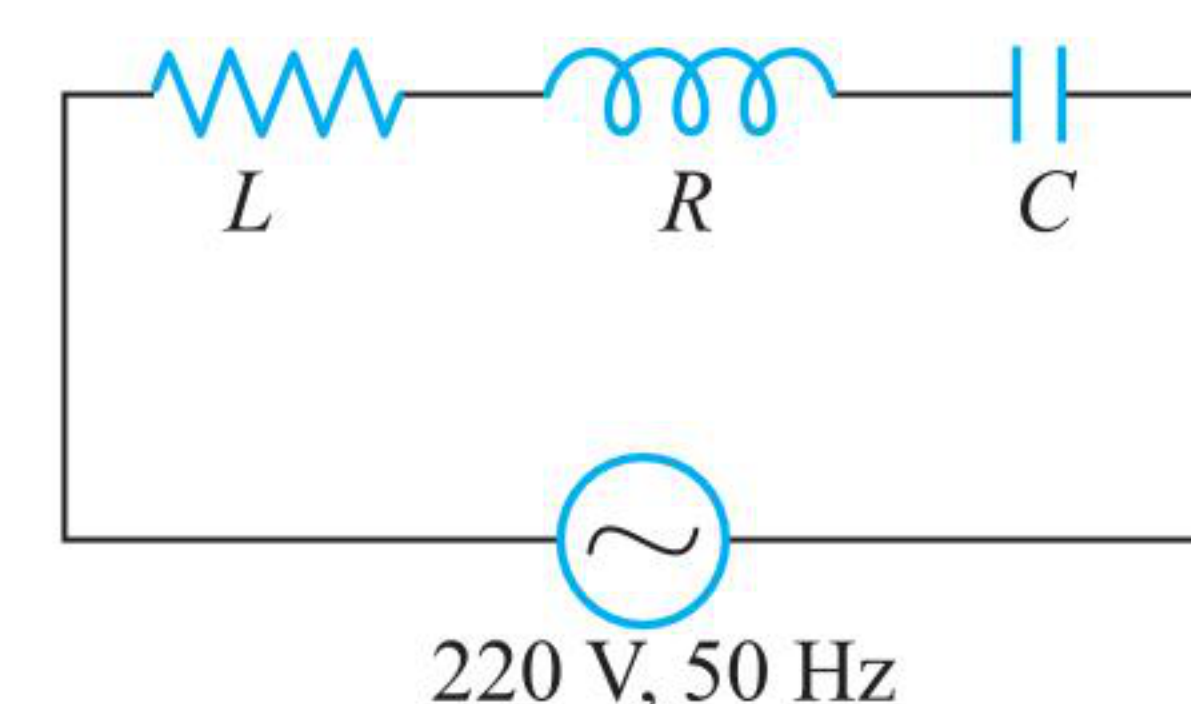
Column I		Column II	
i.		a.	$i_{\text{rms}} = \frac{i_0}{\sqrt{3}}$

ii.		b.	Average current for positive half cycle is i_0
iii.		c.	Average current for positive half cycle is $\frac{i_0}{2}$
iv.		d.	Full cycle average current is zero

3. Four different circuit components are given in each situation of column I and all the components are connected across an ac source of same angular frequency $\omega = 200 \text{ rad s}^{-1}$. The information of phase difference between the current and source voltage in each situation of column I is given in column II. Match the circuit components in column I with corresponding results in column II

Column I		Column II	
i.		a.	The magnitude of required phase difference is $\pi/2$.
ii.		b.	The magnitude of required phase difference is $\pi/4$.
iii.		c.	The current leads in phase to source voltage.
iv.		d.	The current lags in phase to source voltage.

4. In series R - L - C circuit, $R = 100 \Omega$, $C = \frac{100}{\pi} \mu\text{F}$, and $L = \frac{100}{\pi} \text{ mH}$ are connected to an ac source as shown in figure. The rms value of ac voltage is 220 V and its frequency is 50 Hz. In column I, some physical quantities are mentioned, while in column II, information about quantities are provided. Match the entries of column I with the entries of column II.



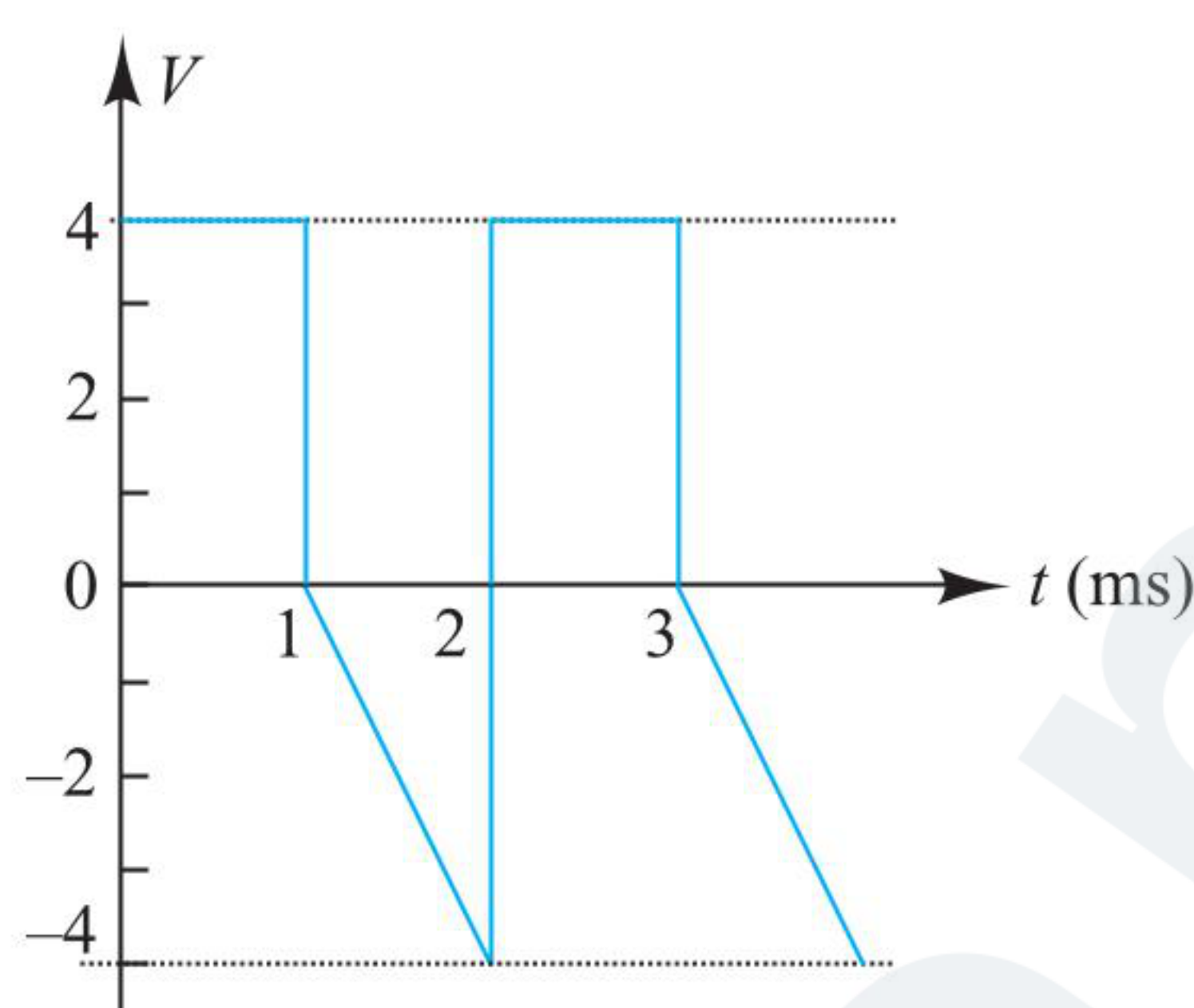
Column I	Column II
i. Average power dissipated in the resistor is	a. zero
ii. Average power dissipated in the inductor is	b. non-zero
iii. Average power dissipated in the capacitor is	c. 163 SI units
iv. The rms voltage across the capacitor is	d. 265.7 SI units

5. Consider all possibilities [L, R, C are non-zero]

Column I	Column II
i. In L - R series ac circuit	a. current lags inductor voltage by $\pi/2$
ii. In R - C series ac circuit	b. current lags voltage by an angle less than $\pi/2$
iii. In L - C - R series ac circuit (consider all possibilities)	c. current leads voltage by an angle less than $\pi/2$
iv. In purely resistive ac circuit	d. current and voltage are in phase

Numerical Value Type

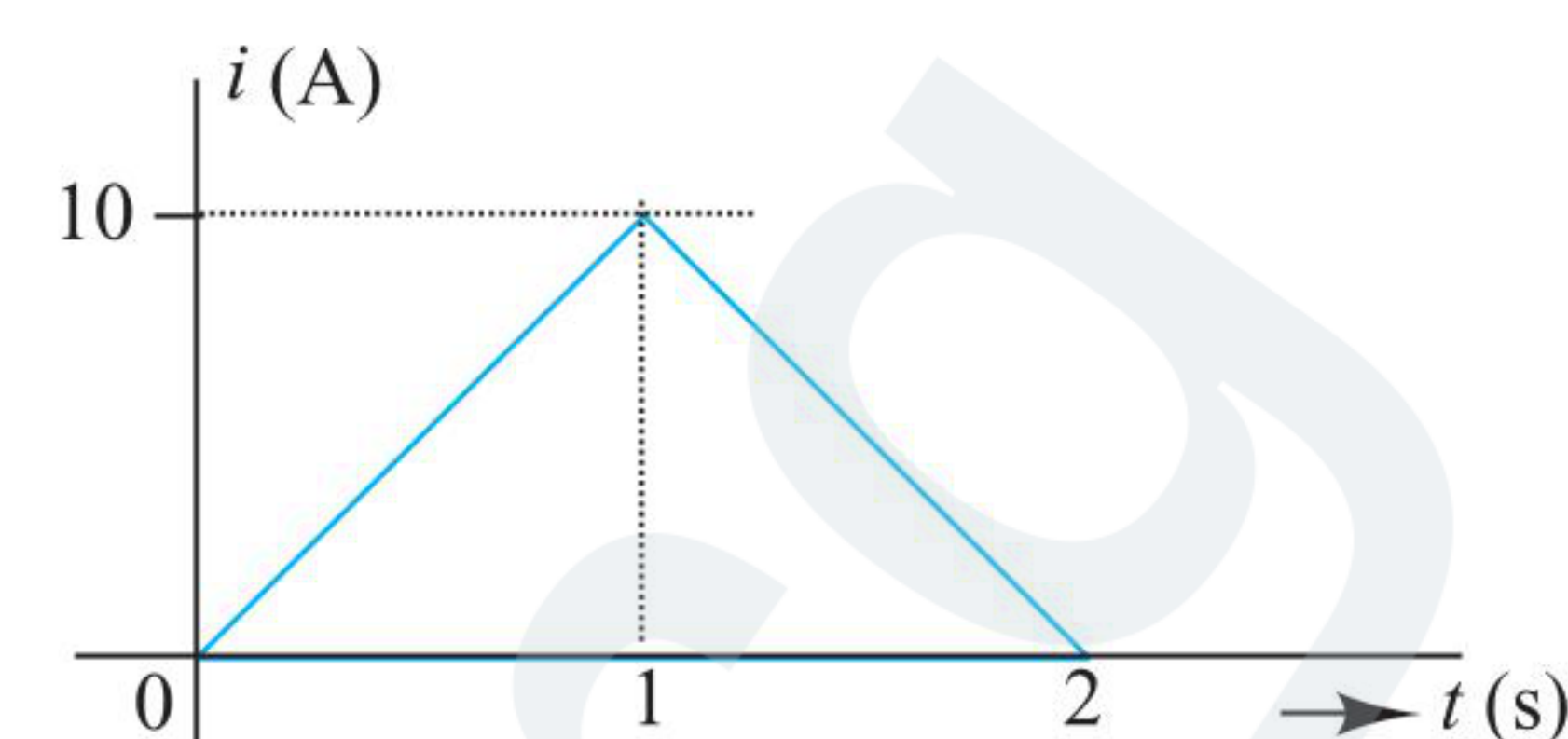
1. Variation of voltage with time is shown in figure.



(a) The rms voltage is found to be $N\sqrt{\frac{2}{3}}$ V. Find N .

(b) Also calculate the average value of voltage (in V).

2. Find the average value of current (in A) shown graphically in figure, from $t = 0$ to $t = 2$ s.

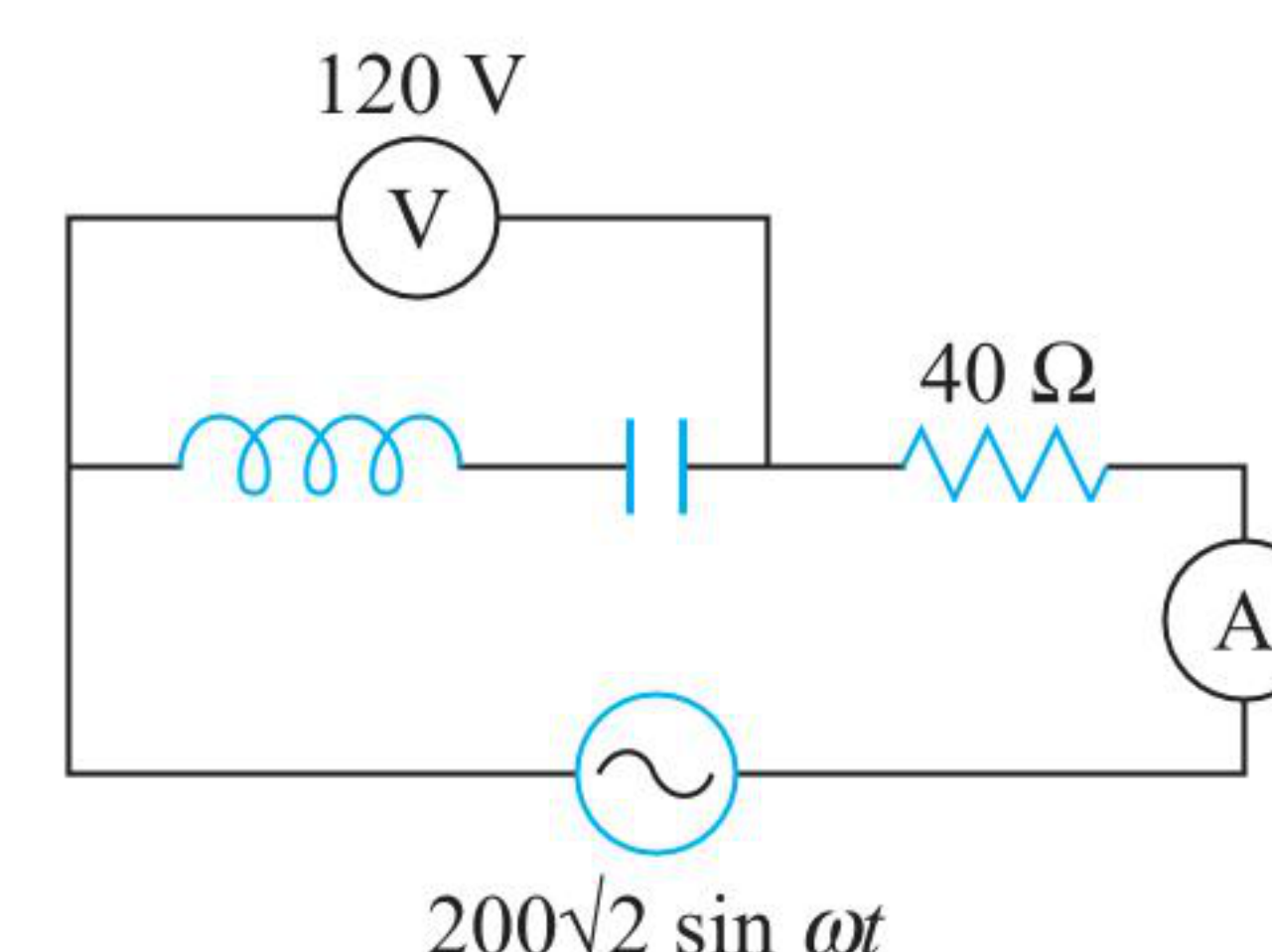


3. The average value of current $i = I_m \sin \omega t$ from $t = \frac{\pi}{2\omega}$ to $t = \frac{3\pi}{2\omega}$ is how many times of I_m ?

4. An ac ammeter is used to measure current in a circuit. When a given direct current passes through the circuit, the ac ammeter reads 3 A. When another alternating current passes through the circuit, the ac ammeter reads 4 A. Then find the reading of this ammeter (in A), if dc and ac flow through the circuit simultaneously.

5. A coil of inductance $L = 5/8$ H and of resistance $R = 62.8 \Omega$ is connected to the mains alternating voltage of frequency 50 Hz. What can be the capacitance of the capacitor (in μF) connected in series with the coil if the power dissipated has to remain unchanged? Take $\pi^2 = 10$.

6. In the given LCR series circuit find the reading (in A) of the hot wire ammeter. (here all hot wire meters are ideal)



Archives

JEE ADVANCED

Single Correct Answer Type

1. An ac voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

- (1) the bulb glows dimmer
- (2) the bulb glows brighter
- (3) total impedance of the circuit is unchanged
- (4) total impedance of the circuit increases

(IIT JEE 2010)

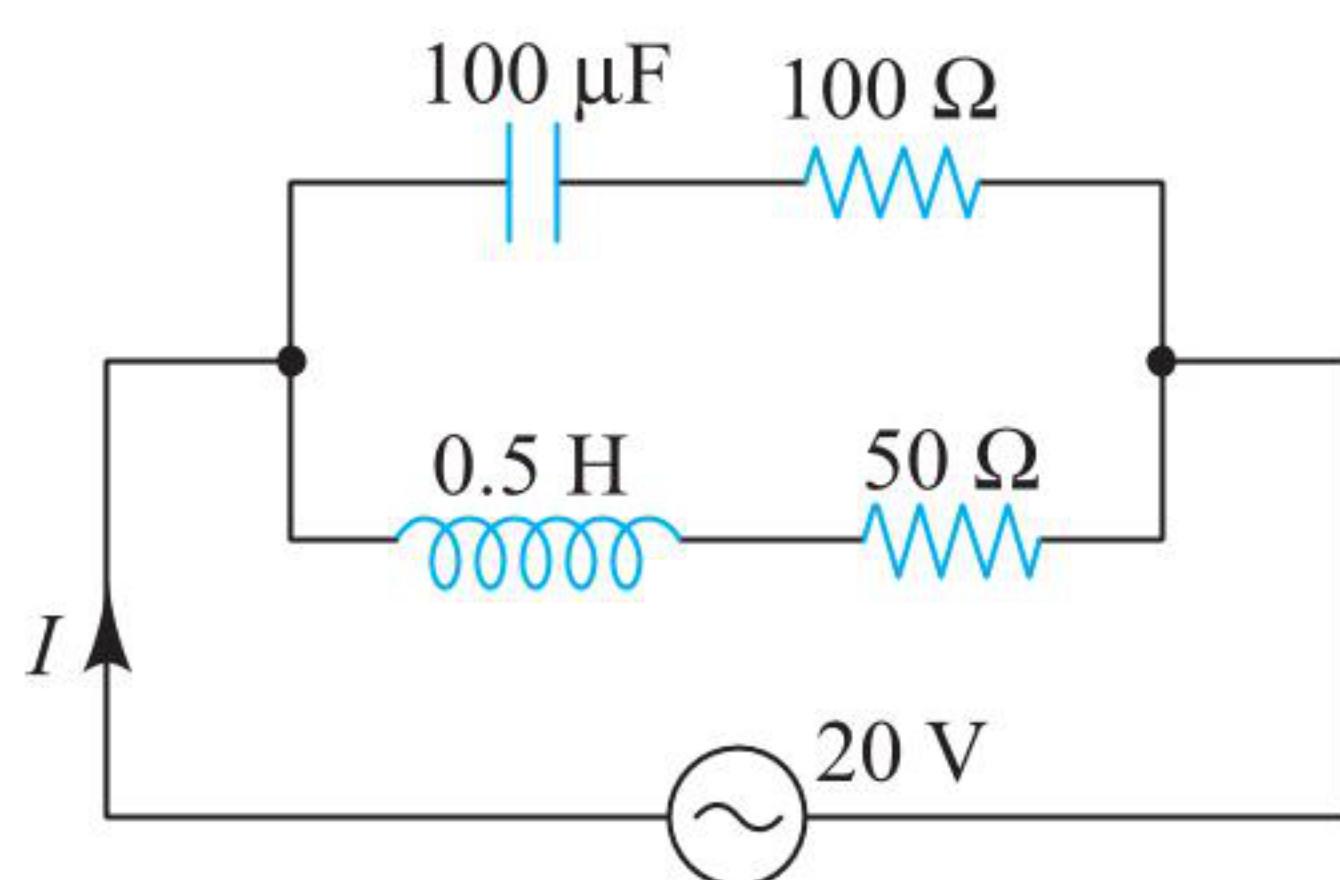
Multiple Correct Answers Type

1. A series R - C circuit is connected to an ac voltage source. Consider two cases: (A) when C is without a dielectric medium and (B) when C is filled with a dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

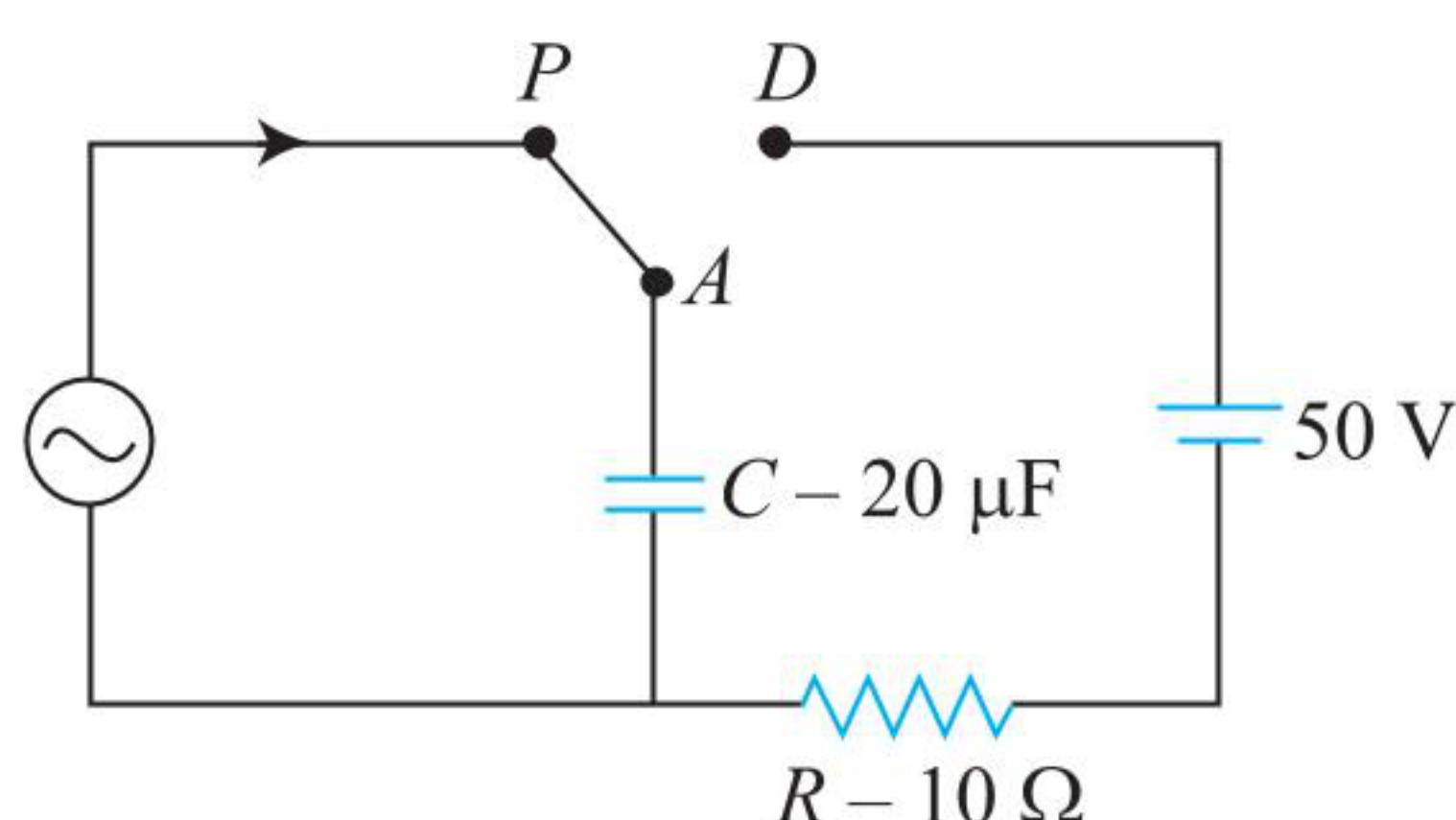
- (1) $I_R^A > I_R^B$
- (2) $I_R^A < I_R^B$
- (3) $V_C^A > V_C^B$
- (4) $V_C^A < V_C^B$

(IIT-JEE 2011)

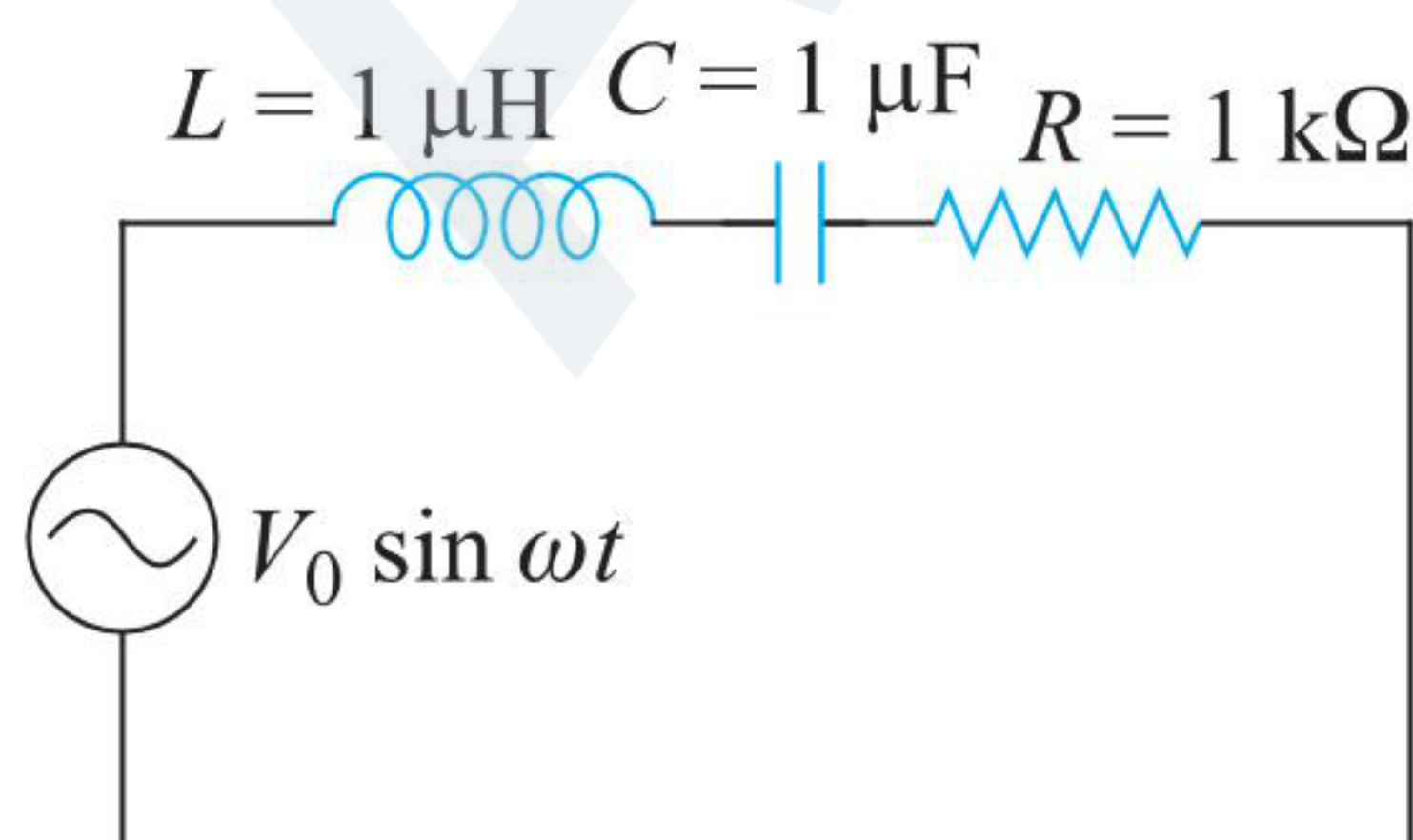
2. In the given circuit, the ac source has $\omega = 100 \text{ rad s}^{-1}$. Considering the inductor and capacitor to be ideal, the correct choice(s) is/are



- (1) The current through the circuit, I is 0.3 A
 (2) The current through the circuit, I is $0.3\sqrt{2}$ A
 (3) The voltage across 100Ω resistor is $10\sqrt{2}$ V
 (4) The voltage across 50Ω resistor is 10 V
- (IIT-JEE 2012)**
3. At time $t = 0$, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0 \cos(\omega t)$, with $I_0 = 1 \text{ A}$ and $\omega = 500 \text{ rad/s}$ starts flowing in it with the initial direction shown in the figure. At $t = 7\pi/6\omega$, the key is switched from B to D . Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20 \mu\text{F}$, $R = 10 \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement(s).



- (1) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is $1 \times 10^{-3} \text{ C}$
 (2) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.
 (3) Immediately after A is connected to D , the current in R is 10 A.
 (4) $Q = 2 \times 10^{-3} \text{ C}$.
- (JEE Advanced 2014)**
4. In the circuit shown, $L = 1 \mu\text{H}$, $C = 1 \mu\text{F}$ and $R = 1 \text{ k}\Omega$. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct?



- (1) At $\omega \sim 0$ the current flowing through the circuit becomes nearly zero
 (2) At $\omega \gg 10^6 \text{ rad s}^{-1}$, the circuit behaves like a capacitor

- (3) The frequency at which the current will be in phase with the voltage is independent of R
 (4) The current will be in phase with the voltage if $\omega = 10^4 \text{ rad s}^{-1}$

(JEE Advanced 2017)

5. The instantaneous voltages at three terminals marked X , Y and Z are given by

$$V_X = V_0 \sin \omega t$$

$$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) \text{ and } V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z . The reading(s) of the voltmeter will be:

(1) $V_{XY}^{rms} = V_0$

(2) $V_{YZ}^{rms} = V_0 \sqrt{\frac{1}{2}}$

- (3) Independent of the choice of the two terminals

(4) $V_{XY}^{rms} = V_0 \sqrt{\frac{3}{2}}$

(JEE Advanced 2017)

Matrix Match Type

1. You are given many resistors, capacitors and inductors. These are connected to a variable dc voltage source (the first two circuits) or an ac voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in Column II. When a current I (steady state for dc or rms for ac) flows through the circuit, the corresponding voltages V_1 and V_2 (indicated in circuits) are related as shown in Column I. Match the two.

Column I		Column II	
i.	$I \neq 0$, V_1 is proportional to I	a.	
ii.	$I \neq 0$, $V_2 > V_1$	b.	
iii.	$V_1 = 0$, $V_2 = V$	c.	
iv.	$I \neq 0$, V_2 is proportional to I	d.	
		e.	

(IIT-JEE 2010)

Numerical Value Type

1. A series R - C combination is connected to an ac voltage of angular frequency $\omega = 500 \text{ rad s}^{-1}$. If the impedance of the R - C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is _____.

(IIT-JEE 2011)

Paragraph for Questions 2 and 3:

In a circuit, a metal filament lamp is connected in series with a capacitor of capacitance $C \text{ }\mu\text{F}$ across a 200 V, 50 Hz supply. The power consumed by the lamp is 500 W while the voltage

drop across it is 100 V. Assume that there is no inductive load in the circuit. Take rms values of the voltages. The magnitude of the phase-angle (in degrees) between the current and the supply voltage is ϕ .

Assume, $\pi\sqrt{3} \approx 5$

2. The value of C is _____.
3. The value of ϕ is _____.

(JEE Advanced 2021)

Answers Key

EXERCISES

Single Correct Answer Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (2) | 2. (1) | 3. (1) | 4. (3) | 5. (2) |
| 6. (2) | 7. (3) | 8. (1) | 9. (3) | 10. (2) |
| 11. (1) | 12. (2) | 13. (4) | 14. (3) | 15. (1) |
| 16. (3) | 17. (1) | 18. (2) | 19. (2) | 20. (2) |
| 21. (1) | 22. (2) | 23. (1) | 24. (1) | 25. (2) |
| 26. (1) | 27. (1) | 28. (3) | 29. (2) | 30. (3) |
| 31. (4) | 32. (4) | 33. (3) | 34. (2) | 35. (2) |
| 36. (2) | 37. (2) | 38. (1) | 39. (2) | 40. (3) |
| 41. (1) | 42. (2) | 43. (3) | 44. (3) | 45. (4) |
| 46. (3) | 47. (1) | 48. (4) | 49. (2) | 50. (3) |
| 51. (1) | 52. (3) | 53. (1) | 54. (4) | 55. (1) |
| 56. (2) | 57. (3) | 58. (3) | 59. (3) | 60. (2) |

Multiple Correct Answers Type

- | | | |
|----------------|----------------|----------------|
| 1. (1),(3) | 2. (1),(2) | 3. (1),(2),(3) |
| 4. (1),(2),(3) | 5. (1),(3),(4) | 6. (1),(3) |
| 7. (2),(3) | 8. (1),(3),(4) | |

Linked Comprehension Type

- | | | | | |
|---------|---------|---------|---------|---------|
| 1. (1) | 2. (2) | 3. (1) | 4. (4) | 5. (1) |
| 6. (3) | 7. (1) | 8. (1) | 9. (1) | 10. (1) |
| 11. (4) | 12. (2) | 13. (4) | 14. (2) | 15. (1) |
| 16. (3) | 17. (1) | 18. (1) | 19. (4) | |

Matrix Match Type

1. i. $\rightarrow$ b., c.; ii. $\rightarrow$ b., c.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ b., c., d.
2. i. $\rightarrow$ d.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ b., d.; iv. $\rightarrow$ b.
3. i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ b., d.
4. i. $\rightarrow$ b., d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b., c.
5. i. $\rightarrow$ a., b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ d.

Numerical Value Type

1. (a) (4), (b) 1 2. (5) 3. (0) 4. (5)
5. (8) 6. (4)

ARCHIVES

JEE Advanced

Single Correct Answer Type

1. (2)

Multiple Correct Answers Type

1. (2),(3) 2. (1),(3) 3. (3),(4) 4. (1),(3) 5. (3),(4)

Matrix Match Type

1. i. $\rightarrow$ c., d., e.; ii. $\rightarrow$ b., c., d., e.; iii. $\rightarrow$ a., b.;
iv. $\rightarrow$ b., c., d., e.

Numerical Value Type

1. (4) 2. (100.00) 3. (60.00)

Chapter 6

Concept Application Exercises

Exercise 6.1

- Yes, because here also (as when 50 Hz), fluctuation in current will be so rapid [30 times/s] that the bulb will appear glowing continuously due to persistence (lesser than 30/s) of vision.
- The brightness will decrease, because L increases on inserting the iron rod and impedance $Z = \sqrt{R^2 + (\omega L)^2}$ increases. This decreases the current, hence brightness decreases.
- The brightness will decrease, because Z increases on increasing ω .
- First decreases, then becomes same. Because as the rod is inserted, L increases. From $f = LI$, I decreases. But soon the current will become same as original.
- $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ decreases on increasing ω . This increases current, hence the brightness will increase.
- $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$ increases on decreasing C . This decreases current, hence the brightness will decrease.
- True, $Z = \sqrt{R^2 + (\omega L)^2} > R$
- If resistance by dc is R , then by ac impedance:

$$Z = \sqrt{R^2 + (\omega L)^2} > R$$

Current will be lesser in case of ac. Hence, heat produced in case of ac will be less.
- False, True

In series current at any time is same in entire circuit. Hence, current is in phase in both L and C . In resistor, voltage is in same phase as current but in inductor, voltage leads current by $\pi/2$. So inductor voltage leads resistance voltage by $\pi/2$.
- False, if frequency of the source is more than resonance frequency then $X_L > X_C$. The circuit becomes inductive. In inductive circuit current lags voltage.
- True, voltage across inductor is given by

$$V_{Lv} = I_v X_L = \frac{E_v X_L}{\sqrt{R^2 + (X_L - X_C)^2}}$$

For certain values of R , X_L , and X_C , X_L can be greater than $Z = \sqrt{R^2 + (X_L - X_C)^2}$. In that case V_{Lv} can be greater than E_v .
- $P = E_v I_v \cos \phi$. If $\cos \phi$ is small, I_v is large for same P .

So loss $= I_v^2 R$ is large.
- True
- 1
- No

Exercises

Single Correct Answer Type

- (2) $E_v = \sqrt{120^2 + 160^2} = 200 \text{ V}$
- (1) The equivalent primary load is

$$R_1 = \left(\frac{N_1}{N_2} \right)^2 R_2 = \left(\frac{20}{1} \right)^2 (6.0) = 2400 \Omega$$

Current in the primary coil

$$= \frac{240}{R_1} = \frac{240}{2400} = 0.1 \text{ A}$$

- (1) Here $V_L = V_C$. They are in opposite phase. Hence, they will cancel each other. Now the resultant potential difference is equal to the applied potential difference = 100 V

$$Z = R \quad (\because X_L = X_C)$$

$$\therefore I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{100}{50} = 2 \text{ A}$$

- (3) According to the given question,

$$\tan 60^\circ = \frac{\omega L}{R} \text{ and } \tan 60^\circ = \frac{1/\omega C}{R}$$

$$\therefore \omega L = (1/\omega C) \text{ (case of resonance)}$$

$$\text{Now } Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2} = 100 \Omega$$

$$\therefore I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{200 \text{ V}}{100 \Omega} = 2 \text{ A}$$

- (2) Given that $E_0 = 10 \text{ V}$, $t = \frac{1}{600} \text{ s}$

$$\begin{aligned} \therefore E &= E_0 \cos 2\pi ft \\ &= 10 \cos \left[2\pi \times 50 \times \frac{1}{600} \right] \\ &= 10 \cos (\pi/6) = 10(\sqrt{3}/2) = 5\sqrt{3} \text{ V} \end{aligned}$$

- (2) Wattless component of ac

$$= I_v \sin \phi = \frac{E_v}{Z} \frac{X_L}{Z} = \frac{E_v X_L}{Z^2} = \frac{220 \times \omega L}{(R^2 + \omega^2 L^2)}$$

$$\text{As } \omega L = 0.7 \times 2\pi \times 50 = 220 \Omega$$

Hence, wattless component of ac

$$= \frac{200 \times (220)}{(220^2 + 220^2)} = 0.5 \text{ A}$$

- (3) Clearly, $X_L = R$

$$\text{or } L \times 2 \times 3.14 \times 1000 = 100$$

$$\text{or } L = \frac{100}{2 \times 3.14 \times 1000} \text{ H}$$

$$= 15.9 \times 10^{-3} \text{ H} = 15.9 \text{ mH} \approx 16 \text{ mH}$$

- (1) Current remains unchanged in R . However, it becomes half in L , because reactance is doubled on doubling the frequency.

- (3) The current of 1.6 A lags emf in phase by $\pi/2$. The current of 0.4 A leads emf in phase by $\pi/2$. So, these two currents are 180° out of phase with each other.

$$\therefore \text{Net current, } I_1 = (1.6 - 0.4) \text{ A} = 1.2 \text{ A}$$

$$1.6 = \frac{E_v}{X_L} \text{ and } 0.4 = \frac{E_v}{X_C}$$

$$\Rightarrow \frac{X_C}{X_L} = 4 \Rightarrow \frac{1}{\omega C \omega L} = 4$$

$$\Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

$$\Rightarrow f = \frac{\omega}{2\pi} = \frac{1}{4\pi\sqrt{LC}}$$

$$10. (2) X_C = \frac{1}{2\pi fC} = \frac{10^6}{2\pi \times 50 \times 500} = \frac{20}{11} \Omega$$

$$X_L = 2\pi fL = 2\pi \times 50 \times 10 \times 10^{-3} = \pi \Omega$$

Since $X_L < X_C$, so inductive branch has less impedance, and so more current. Hence B_2 will be brighter.

11. (1) Since the capacitor is connected in series to the resistor, current I_C from the supply and I_R through the resistor is in phase as represented by choice (1).

12. (2) Root mean square current of the sinusoidal waveform, $I = \frac{I_0}{\sqrt{2}}$.

Power output of the heater,

$$P = I^2 R = \left(\frac{I_0}{\sqrt{2}} \right)^2 R = \frac{I_0^2 R}{2}$$

13. (4) Both V and I are in the same phase. So, let us calculate the time taken by the voltage to change from peak value to rms value. Now $220 = 220 \sin 100\pi t_1$

$$\text{or } 100\pi t_1 = \frac{\pi}{2} \quad \text{or } t_1 = \frac{1}{200} \text{ s}$$

$$\text{Again, } \frac{220}{\sqrt{2}} = 220 \sin 100\pi t_2$$

$$\text{or } \frac{1}{\sqrt{2}} = \sin 100\pi t_2 \quad \text{or } 100\pi t_2 = \frac{3\pi}{4}$$

$$\text{or } t_2 = \frac{3}{400} \text{ s}$$

$$\text{Required time} = t_2 - t_1 = \frac{1}{400} \text{ s} = 2.5 \times 10^{-3} \text{ s}$$

14. (3) Quality factor

$$= \frac{f_0}{f_2 - f_1} = \frac{600}{650 - 550} = \frac{600}{100} = 6$$

15. (1) $E = E_0 \cos \omega t$

$$\therefore \omega = 50\pi$$

$$2\pi f = 50\pi \Rightarrow f = 25 \text{ Hz}$$

In one cycle ac current becomes zero twice.

Therefore, 50 times the current becomes zero in 1 s.

16. (3) $\lambda = 300 \text{ m}$, $c = 3 \times 10^8 \text{ m s}^{-1}$

$$\text{Frequency } f = \frac{c}{\lambda} = \frac{3 \times 10^8}{300} = 10^6 \text{ Hz}$$

Resonance frequency

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{or } L = \frac{1}{C4\pi^2 f^2}$$

$$\therefore L = \frac{1}{4\pi^2 (10^6)^2 \times 2.4 \times 10^{-6}} \equiv 10^{-8} \text{ H}$$

17. (1) Change on the capacitor,

$$q_0 = CV = 1 \times 10^{-6} \times 1 = 10^{-6} \text{ C}$$

or $I_0 = \omega q_0$ = Maximum current

$$\text{Now } \omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-9}}} = (10^9)^{1/2}$$

$$\therefore I_0 = (10^9)^{1/2} \times (1 \times 10^{-6}) = \sqrt{10^{-3}} \text{ A} = \sqrt{1000} \text{ mA}$$

18. (2) $E = E_0 \sin \omega t$

Voltage read is rms value.

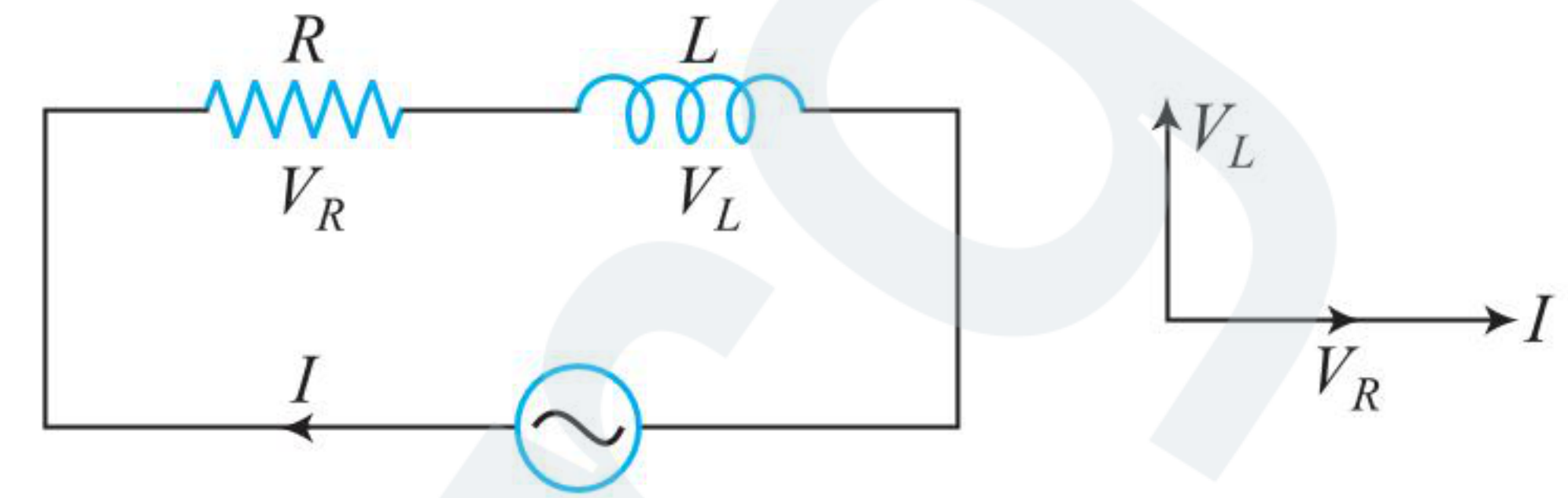
$$\therefore E_0 = \sqrt{2} \times 234 \text{ V} = 331 \text{ V}$$

$$\text{and } \omega t = 2\pi f t = 2\pi \times 50 \times t = 100\pi t$$

Thus, the equation of the line voltage is given by

$$E = 331 \sin(100\pi t)$$

19. (2) Here inductance and resistance are connected in series. We know that in case of resistance, both current and potential difference are in the same phase. In inductor, voltage leads current by $\pi/2$.



20. (2) Here $R = X_L = X_C$ ($\therefore$ voltage across them is same)
When capacitor is short circuited,

$$I = \frac{10}{(R^2 + X_L^2)^{1/2}} = \frac{10}{\sqrt{2}R}$$

$$\therefore \text{Potential drop across inductance} = IX_L = IR = 10/\sqrt{2} \text{ V}$$

21. (1) $R = \frac{125}{12.5} = 10 \Omega$

$$X_L = \omega L = 2\pi fL = \frac{V}{I} = \frac{125}{10} = 12.5$$

$$\therefore X'_L = 2\pi L \times f' = 0.25 \times 40 = 10 \Omega$$

Impedance of the circuit

$$Z = \sqrt{R^2 + X_L^2} = 10\sqrt{2} \Omega$$

$$\therefore \text{Current} = \frac{100}{10\sqrt{2}} = 10/\sqrt{2} \text{ A}$$

22. (2) Given $I = 5 + 10 \sin \omega t$

$$I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt} \right]^{1/2} = \left[\frac{1}{T} \int_0^T (5 + 10 \sin \omega t)^2 dt \right]^{1/2}$$

$$= \left[\frac{1}{T} \int_0^T (25 + 100 \sin \omega t + 100 \sin^2 \omega t) dt \right]^{1/2}$$

$$\text{But as } \frac{1}{T} \int_0^T \sin \omega t dt = 0$$

$$\text{and } \frac{1}{T} \int_0^T \sin^2 \omega t dt = \frac{1}{2}$$

$$\text{so } I_{\text{eff}} = \left[25 + \frac{1}{2} \times 100 \right]^{1/2} = 5\sqrt{3} \text{ A}$$

23. (1) $X_L = L\omega = 5 \times 10^{-3} \times 2000 = 10 \Omega$

$$X_C = \frac{1}{C\omega} = \frac{1}{50 \times 10^{-6} \times 2000} = \frac{100}{10} \Omega = 10 \Omega$$

Since $X_L = X_C$, therefore,

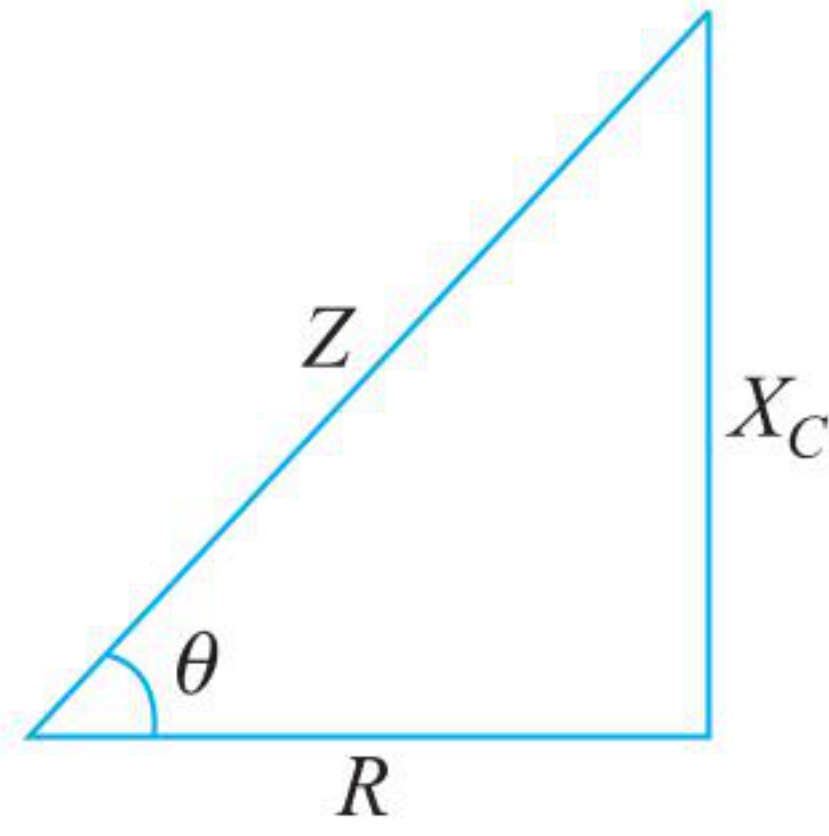
$$Z = R = 5.9 + 0.10 + 4 = 10 \Omega$$

$$I_v = \frac{E_v}{Z} = \frac{20}{10} \text{ A} = 2 \text{ A}$$

24. (1) $P = E_v I_v \cos \phi$; $P = E_v \frac{E_v}{Z} \frac{R}{Z}$

$$\text{or } P = \frac{E_v^2 R}{Z^2} = \frac{110 \times 110 \times 11}{22 \times 22} = 275 \text{ W}$$

25. (2) $V_C^2 + V_R^2 = V^2$



$$50^2 + V_R^2 = 110^2 \Rightarrow V_R = \sqrt{160 \times 60} = 98 \text{ V}$$

$$\text{Then, } I_V = \frac{\sqrt{160 \times 60}}{50} \quad (\because R = 50 \Omega)$$

$$\text{Also, } I_V = \frac{110}{\sqrt{R^2 + X_C^2}} \therefore \frac{98}{50} = \frac{110}{\sqrt{50^2 + X_C^2}}$$

$$\text{Flux } X_C \text{ and now using } X_C = \frac{1}{\omega C}, \text{ we get } C = 104 \mu\text{F}$$

26. (1) $q_0 = CE_0 = 8 \times 10^{-6} \times 220 \sqrt{2} = 2.5 \times 10^{-3} \text{ C}$

27. (1) Resultant current is superposition of two currents, i.e.,
 I (instantaneous total current) $= 6 + I_0 \sin \omega t$
 dc ammeter will read average value

$$= \overline{6 + I_0 \sin \omega t} = 6 \quad (\because \overline{I_0 \sin \omega t} = 0)$$

ac ammeter will read

$$= \sqrt{\overline{(6 + I_0 \sin \omega t)^2}}$$

$$= \sqrt{\overline{36 + 12I_0 \sin \omega t + I_0^2 \sin^2 \omega t}} \quad (\because \overline{I_0 \sin \omega t} = 0)$$

$$\text{Since } \overline{\sin^2 \omega t} = \frac{1}{2} \text{ and } I_{\text{rms}} = 8 = \frac{I_0}{\sqrt{2}} \Rightarrow I_0 = 8\sqrt{2} \text{ A}$$

Therefore, ac reading

$$= \sqrt{36 + \frac{I_0^2}{2}} = \sqrt{36 + 64} = 10 \text{ A}$$

28. (3) $I = I_1 \cos \omega t + I_2 \sin \omega t$

$$(I^2)_{\text{mean}} = \overline{I_1^2 \cos^2 \omega t + I_2^2 \sin^2 \omega t + 2I_1 I_2 \cos \omega t \sin \omega t}$$

$$= I_1^2 \times \frac{1}{2} + I_2^2 \times \frac{1}{2} + 2I_1 I_2 \times 0$$

$$I_{\text{rms}} = \sqrt{(I^2)_{\text{mean}}} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

29. (2) Resistance of bulb: $R = \frac{V_0^2}{P_0} = \frac{(220)^2}{1100} = 44 \Omega$

when L is maximum, power consumed will be minimum which

$$\text{is } P = \frac{1100}{5} = 220 \text{ W}$$

$$\text{Now } P = I_V^2 R \Rightarrow 220 = I_V^2 \times 44 \Rightarrow I_V = \sqrt{5} \text{ A}$$

$$I_V = \frac{E_V}{Z} \Rightarrow \sqrt{5} = \frac{220}{\sqrt{44^2 + X_L^2}} \Rightarrow X_L = 88 \Omega$$

$$\Rightarrow 2\pi f L_{\text{max}} = 88 \Rightarrow 2 \times \frac{22}{7} \times 50 L_{\text{max}} = 88$$

$$\Rightarrow L_{\text{max}} = 0.28 \text{ H}$$

30. (3) $E_1 = E_0 \sin \omega t$; $E_2 = E_0 \sin[\omega t + (\pi/3)]$

$$E = E_2 + E_1$$

$$= E_0 \sin[\omega t + (\pi/3)] + E_0 \sin \omega t$$

$$= E_0 [2 \sin\{\omega t + (\pi/6)\} \cos(\pi/6)]$$

$$= \sqrt{3} E_0 \sin[\omega t + (\pi/6)]$$

31. (4) If an ac source $E = E_0 \sin \omega t$ is applied across an inductance and capacitance in parallel, the current in inductance will lag the applied voltage while that across the capacitor will lead, and so,

$$I_L = \frac{E_0}{X_L} \sin\left(\omega t - \frac{\pi}{2}\right) = -0.8\sqrt{2} \cos \omega t$$

$$I_C = \frac{V}{X_C} \sin\left(\omega t + \frac{\pi}{2}\right) = 0.6\sqrt{2} \cos \omega t$$

So the current drawn from the source

$$I = I_L + I_C = -0.2\sqrt{2} \cos \omega t$$

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}} = \frac{0.2\sqrt{2}}{\sqrt{2}} = 0.2 \text{ A}$$

32. (4) As current at any instant in the circuit will be

$$I = I_{\text{dc}} + I_{\text{ac}} = a + b \sin \omega t$$

$$\text{so, } I_{\text{eff}} = \left[\frac{\int_0^T I^2 dt}{\int_0^T dt} \right]^{1/2} = \left[\frac{1}{T} \int_0^T (a + b \sin \omega t)^2 dt \right]^{1/2}$$

$$\text{i.e., } I_{\text{eff}} = \left[\frac{1}{T} \int_0^T (a^2 + 2ab \sin \omega t + b^2 \sin^2 \omega t) dt \right]^{1/2}$$

$$\text{but as } \frac{1}{T} \int_0^T \sin \omega t dt = 0 \quad \text{and} \quad \frac{1}{T} \int_0^T \sin^2 \omega t dt = \frac{1}{2}$$

$$\text{So, } I_{\text{eff}} = \left[a^2 + \frac{1}{2} b^2 \right]^{1/2}$$

33. (3) The equation of a semi-circular wave is

$$x^2 + y^2 = a^2 \quad \text{or} \quad y^2 = a^2 - x^2$$

$$I_{\text{rms}} = \sqrt{\frac{1}{2a} \int_{-a}^{+a} y^2 dx}$$

$$I_{\text{rms}}^2 = \frac{1}{2a} \int_{-a}^{+a} (a^2 - x^2) dx$$

$$= \frac{1}{2a} \int_{-a}^{+a} (a^2 - x^2) dx = \frac{1}{2a} \left[a^2 x - \frac{x^3}{3} \right]_{-a}^{+a}$$

$$= \frac{1}{2a} \left(a^3 - \frac{a^3}{3} + a^3 - \frac{a^3}{3} \right) = \frac{2a^2}{3}$$

$$I_{\text{rms}} = \sqrt{\frac{2a^2}{3}} = \sqrt{\frac{2}{3}} a$$

34. (2) $X_C = \frac{1}{\omega C} = \frac{1}{100 \times 10^{-6}} = 10^4 \Omega$

$$\text{So, } I_{\text{rms}} = \frac{E_{\text{rms}}}{Z} = \frac{50\sqrt{2}}{\sqrt{2} \times 10^4} = 5 \text{ mA}$$

35. (2) At resonance, the series combination of L and C gives zero impedance.

At resonance, the voltages across L and C are equal but opposite in phase.

36. (2) As resistance of the lamp

$$R = \frac{V_S^2}{P_0} = \frac{100^2}{50} = 200 \Omega$$

The rms current $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}$ A.

So when the lamp is put in series with a capacitance and run at 200 V ac, from $V = IZ$, we have

$$Z = \frac{V}{I} = \frac{200}{(1/2)} = 400 \Omega$$

Now as in case of C - R circuit,

$$Z = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\text{i.e., } R^2 + \left(\frac{1}{\omega C}\right)^2 = 160000$$

$$\text{or, } \left(\frac{1}{\omega C}\right)^2 = 16 \times 10^4 - (200)^2 = 12 \times 10^4$$

$$\frac{1}{\omega C} = \sqrt{12} \times 10^2$$

$$C = \frac{1}{100\pi \times \sqrt{12} \times 10^2} \text{ F}$$

$$= \frac{100}{\pi\sqrt{12}} \mu\text{F} = \frac{50}{\pi\sqrt{3}} = 9.2 \mu\text{F}$$

37. (2) Here, $X_L = \omega L = 2\pi fL = 2\pi \times 50 \times 1 = 100 \pi \Omega$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = \frac{10^3}{\pi} \Omega$$

$$\text{So, } X = |X_L - X_C| = \left| 100\pi - \frac{10^3}{\pi} \right| = 10^2 \left[\frac{\pi^2 - 10}{\pi} \right] \Omega$$

38. (1) $i_3 = i_1 + i_2 = 3 \sin \omega t + 4 \cos \omega t$
 $= 3 \sin \omega t + 4 \sin(\omega t + 90^\circ)$

$$i_{30} = \sqrt{i_{10}^2 + i_{20}^2} = \sqrt{3^2 + 4^2} = 5$$

$$\tan \phi = \frac{4}{3} \Rightarrow \phi = 53^\circ$$

$$\text{So } i_3 = i_{30} \sin(\omega t + \phi) = 5 \sin(\omega t + 53^\circ)$$

39. (2) $V_{\text{rms}} = \sqrt{16^2 + 20^2} = 25.6 \text{ V}$

40. (3) $i = 3 \sin \omega t + 4 \cos \omega t$

$$= 5 \left[\frac{3}{5} \sin \omega t + \frac{4}{5} \cos \omega t \right] = 5[\sin(\omega t + \delta)] \quad \dots(i)$$

$$\text{rms value} = \frac{5}{\sqrt{2}} \text{ A}$$

If voltage applied is $V = V_m \sin \omega t$ then I , given by Eq. (i), indicates that it is ahead of V by δ where $0 < \delta < 90$ which indicates that the circuit contains R and C .

If we find average value for the first half period (0 to $T/2$), then it will be $6/\pi$ A. But for different time interval, it will be different.

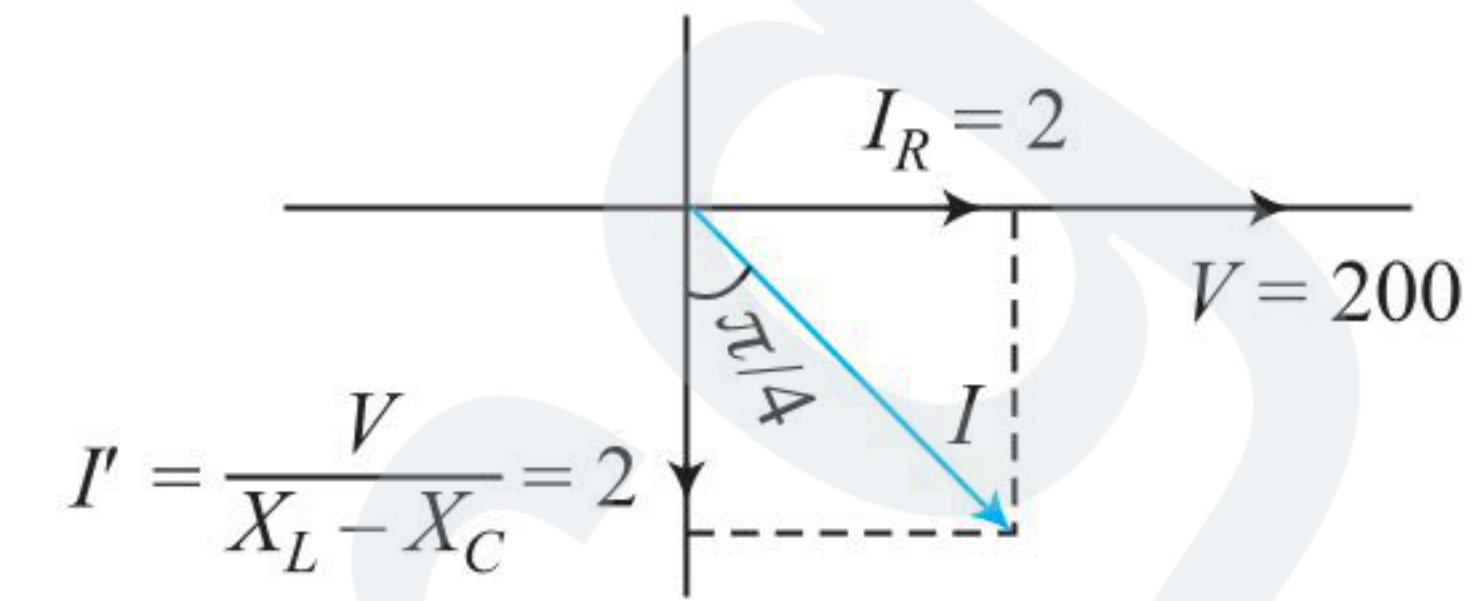
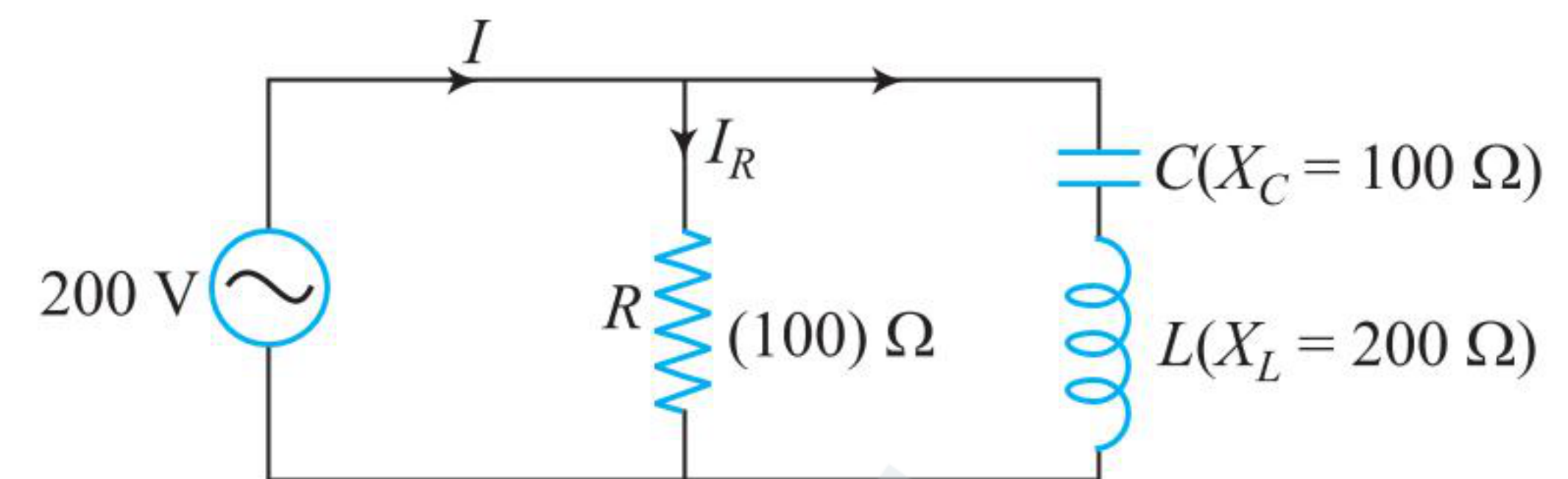
41. (1) $v = v_0 \sin[\omega t + (\pi/4)] = v_0 \cos[\omega t - (\pi/4)]$

Since V lags current, an inductor can bring it in phase with current.

42. (2) $I_R = \frac{V}{R} = \frac{200}{100} = 2 \text{ A}$

$$I' = \frac{V}{X_L - X_C} = \frac{200}{100} = 2 \text{ A}$$

$$I = \sqrt{I_R^2 + I'^2} = 2\sqrt{2} \text{ A}$$



43. (3) The circuit will have inductive nature if

$$\omega > \frac{1}{\sqrt{LC}} \left(\omega L > \frac{1}{\sqrt{LC}} \right)$$

Hence (1) is false. Also, if circuit has inductive nature, the current will lag behind voltage. Hence, (4) is also false.

If $\omega = \frac{1}{\sqrt{LC}}$ ($\omega L = \frac{1}{\omega C}$), the circuit will have resistance nature. Hence, (2) is false.

$$\text{Power factor, } \cos \phi = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C} \right)^2}} = 1$$

If $\omega L = \frac{1}{\omega C}$. Hence, (3) is true.

44. (3) $X_L = X_C$ at resonance

$$\frac{X_L}{X_C} = 1 \text{ for both circuits.}$$

Impedance may be different if applied voltage is different.

45. (4) Since $\cos \phi = \frac{R}{Z} = \frac{IR}{IZ} = \frac{8}{10} = \frac{4}{5}$

(Also $\cos \phi$ can never be greater than 1)

Hence, (c) is wrong.

Also $IX_C > IX_L \Rightarrow X_C > X_L$

$\therefore$ Current will be leading.

In an LCR circuit,

$$V = \sqrt{(v_L - v_C)^2 + v_R^2} = \sqrt{(6 - 12)^2 + 8^2}$$

$V = 10$; which is less than voltage drop across capacitor.

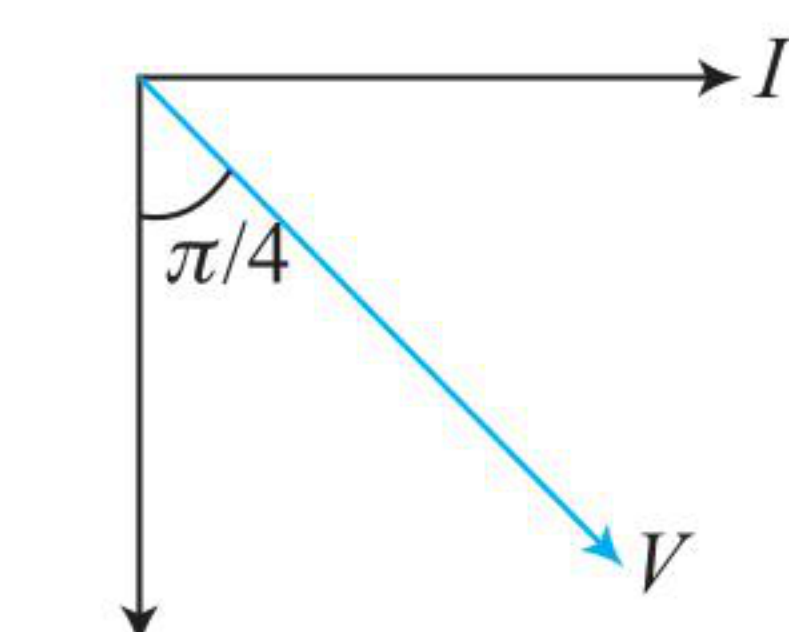
46. (3) Since current is leading emf, capacitor must be present in the circuit. Along with capacitor, there can be either (i) only resistor or (ii) resistor and inductor if $X_C > X_L$.

In both of the above conditions, we can obtain a phase difference of $\pi/4$ with current leading emf.

47. (1) $V = \frac{V_0}{T/4} t; V = \frac{4V_0}{T} t$

$$V_{\text{rms}} = \sqrt{\langle V^2 \rangle} = \frac{4V_0}{T} \sqrt{\langle t^2 \rangle}$$

$$= \frac{4V_0}{T} \left\{ \frac{\int_0^{T/4} t^2 dt}{\int_0^{T/4} dt} \right\}^{1/2} = \frac{V_0}{\sqrt{3}}$$



$$48. (4) \langle V \rangle = \frac{\int_0^T V dt}{\int_0^T dt} = 0$$

$$49. (2) (0.5)^2 R \left(\frac{T}{2} \right) + (0.5)^2 R \left(\frac{T}{2} \right) = I_{\text{rms}}^2 RT$$

$$\text{or } I_{\text{rms}} = \frac{1}{2} \text{ A} = 0.5 \text{ A}$$

$$50. (3) \cos \phi = \frac{R}{\sqrt{R^2 + \left(\frac{1}{\omega C} - \omega L \right)^2}}$$

Putting the values, $C = 500 \mu\text{F}$

51. (1) Given:

Voltage in primary coil, $V_p = 200 \text{ V}$

Current in primary coil, $i_p = 2 \text{ A}$

Voltage in secondary coil, $V_s = 2000 \text{ V}$

The relation for the current in the secondary coil is

$$\frac{V_s}{V_p} = \frac{i_p}{i_s} \Rightarrow \frac{2000}{200} = \frac{2}{i_s} = \frac{2 \times 200}{2000} = 0.2 \text{ A}$$

52. (3) $P_{\text{av}} = E_v I_v \cos \phi = 100 \times 10 \cos \phi = 1000 \cos \phi \text{ Watt}$
So depending upon value of ϕ , P_{av} can be equal to or less than 1000 W.

53. (1) $E = 120 \sin 100 \pi t \cos 100 \pi t = 60 \sin (200 \pi t)$
So peak voltage $E_0 = 60 \text{ V}$

54. (4) Both B and I are in same phase. So, let us calculate the time taken by the voltage to change from peak value of rms value.
Now, $220 = 220 \sin 100 \pi t_1$

$$\text{or } 100 \pi t_1 = \frac{\pi}{2} \quad \text{or } t_1 = \frac{1}{200} \text{ s}$$

$$\text{Again, } \frac{200}{\sqrt{2}} = 200 \sin 100 \pi t_2$$

$$\text{or } \frac{1}{\sqrt{2}} = \sin 100 \pi t_2 \quad \text{or } 100 \pi t_2 = \frac{\pi}{4}$$

$$\text{or } t_2 = \frac{1}{400} \text{ s}$$

Required time $= t_1 - t_2$.

$$= \frac{1}{200} - \frac{1}{400} = \frac{2-1}{400} = \frac{1}{400} \text{ s} = 2.5 \times 10^{-3} \text{ s}$$

$$55. (1) I_{\text{rms}}^2 T = \int_0^T (3 + 4 \sin \omega t)^2 dt$$

$$56. (2) \tan \theta = \frac{X_L}{R} = 1 \quad \therefore \theta = 45^\circ$$

$$t = \theta \times \frac{\text{Time period}}{360}$$

57. (3) For LR circuit, $XL = \omega L$.

$$\tan \phi = \frac{\omega L}{R} = \frac{X_L}{R}$$

$$\text{For } \phi = 45^\circ, \tan 45^\circ = \frac{X_L}{R} \quad \text{or } X_L = R$$

$$58. (3) H(\text{DC}) = I^2 R t = 2^2 R t \text{ joule}$$

$$H(\text{AC}) = I^2 R t = \left(\frac{I_0}{\sqrt{2}} \right)^2 R t = \frac{2^2}{2} R t$$

$$\therefore \frac{H(\text{DC})}{H(\text{AC})} = \frac{2}{1}$$

59. (3) Voltage across L and C cancel out.

So, voltage across k is 220 V.

$$\text{Again, } I_0 = \frac{220}{100} \text{ A} = 2.2 \text{ A}$$

60. (2) Let I_1 and I_2 be currents through B_1 and B_2 then

$$I_1 \times \sqrt{R^2 + X_L^2} = 220$$

$$\frac{I_2}{I_1} = \frac{\sqrt{R^2 + X_C^2}}{\sqrt{R^2 + X_L^2}} = \frac{\sqrt{R^2 + \left(\frac{1}{\omega C} \right)^2}}{\sqrt{R^2 + L^2 \omega^2}}$$

$$= \frac{\sqrt{R^2 + \left(\frac{1}{500 \times 10^{-6} \times 2\pi \times 50} \right)^2}}{\sqrt{R^2 + (10 \times 10^{-3} \times 2\pi \times 50)^2}}$$

$$= \sqrt{R^2 + 40} / \sqrt{R^2 + 9.87}, I_2 > I_1$$

Bulb B_2 will be brighter. As frequency increases, X_C decreases X_L increases. I_2 becomes less and I_1 increases.

$\therefore$ brightness of B_1 will increase and that of B_2 decreases.

Multiple Correct Answers Type

1. (1), (3)

$$f_{\text{res}} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi} \times \frac{1}{\pi} \times 10^{-6}}} = 500 \text{ Hz}$$

At resonance, $Z = R$,

$$\text{So current } I = \frac{V}{Z} = \frac{V}{R}$$

When L and C are in series, voltage across capacitor and inductor is 180° out of phase.

2. (1), (2)

($f = 900 \text{ Hz}$) < ($f_{\text{res}} = 1000 \text{ Hz}$), so $X_C > X_L$, circuit becomes capacitive and current leads emf.

At resonance, impedance is minimum.

Not only at resonance, but at all conditions voltage across L and C differs in phase by 180° when connected in series.

3. (1), (2), (3)

$H = I^2 R t / 4.2$ (Heat produced in a conductor is independent of the nature of current. If the current flowing in a conductor changes its direction, then also heat being produced in it will not change). Hence, (1), (2), and (3) are true.

4. (1), (2), (3)

The frequency

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.6 \times 10 \times 10^{-6}}} = 65 \text{ Hz}$$

The current at resonance is

$$I_0 = \frac{200 \text{ V}}{5 \Omega} = 40 \text{ A}$$

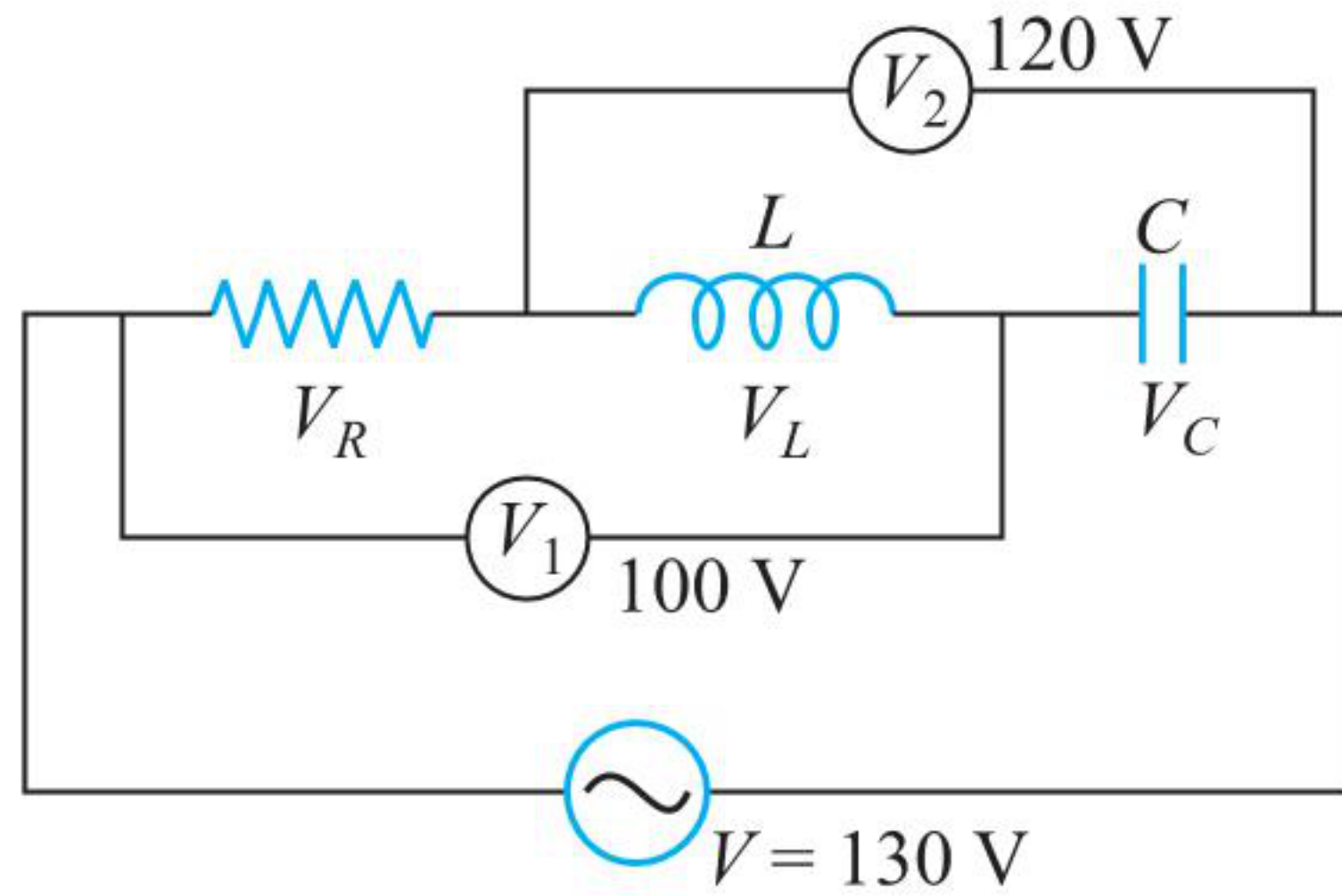
$$\text{The quantity } V_0 = I_0 \sqrt{R^2 + (\omega L)^2}$$

$$= 40\sqrt{25^2 + (0.6 \times 2\pi \times 65)^2} = 9.8 \text{ kV}$$

$$V_1 = \frac{I_0}{\omega C} = \frac{40 \times 10^5}{2\pi \times 65} = 9.8 \text{ kV}$$

5. (1), (3), (4)

$$V_R^2 + V_L^2 = 100^2, |V_C - V_L| = 120$$



$$130^2 = V_R^2 + (V_C - V_L)^2 \Rightarrow V_R = 50 \text{ V}$$

$$\Rightarrow V_L = \sqrt{100^2 - 50^2} = 50\sqrt{3} \text{ V}$$

$$V_C = 120 + 50\sqrt{3} \text{ V}$$

$$\cos \phi = \frac{V_R}{V} = \frac{50}{130} = \frac{5}{13}$$

Since $V_C < V_L$, so circuit is capacitive.

6. (1), (3)

$$\text{average current} = \frac{\text{total charge}}{\text{time}}$$

Total charge will be equal to area of $I-t$ graph, which is zero for $T/2$ to $3T/2$, hence average current will be zero for this time interval. for $T/2$ to $3T/2$:

$$I = \frac{2I_0 t}{T} - 2I_0$$

$$I_{\text{rms}} = \sqrt{\frac{\int_{T/2}^{3T/2} I^2 dt}{3T/2 - T/2}} = I_0/\sqrt{3}$$

7. (2), (3)

$$I = I_C + I_L$$

$$\Rightarrow I = I = \frac{E_0}{X_C} \sin(\omega t + \pi/2) + \frac{E_0}{X_L} \sin(\omega t - \pi/2)$$

$$= \left[\frac{E_0}{X_C} - \frac{E_0}{X_L} \right] \cos \omega t$$

$$\text{To find: } I_v = \left| \frac{1}{\sqrt{2}} \left[\frac{E_0}{X_C} - \frac{E_0}{X_L} \right] \right|$$

$$\text{Given: } I_{Cv} = \frac{E_0}{\sqrt{2}X_C} = 0.4 \text{ A}$$

$$I_{Lv} = \frac{E_0}{\sqrt{2}X_L} = 1.6 \text{ A}$$

From equations (i), (ii) and (iii) $I_v = 1.2 \text{ A}$

Dividing (ii) by (iii)

$$\frac{X_L}{X_C} = \frac{1}{4} \Rightarrow \frac{\omega L}{1/\omega C} = \frac{1}{4} \Rightarrow \omega^2 LC = \frac{1}{4} \Rightarrow \omega = \frac{1}{2\sqrt{LC}}$$

8. (1), (3), (4)

$$V_1 = V_2 \Rightarrow X_L = X_C$$

$$\Rightarrow f = \frac{1}{2\pi\sqrt{LC}} = 125 \text{ Hz}$$

$$I_{\text{rms}} = \frac{V}{R} = \frac{200}{100} = 2 \text{ A} \quad (\because Z = R)$$

$$V_1 = V_2 = IXL = I(\omega L) = 1000 \text{ volt}$$

Linked Comprehension Type

For Problems 1–3

1. (1) 2. (2) 3. (1)

1. (1) The peak value, $V_0 = 220\sqrt{2} = 311 \text{ V}$

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}} = \frac{220\sqrt{2}}{\sqrt{2}} = 200 \text{ V}$$

$$2. (2) V_{\text{av}} = \frac{2}{\pi} V_0 = \frac{2}{\pi} \times 311 = \frac{622}{\pi} \text{ V}$$

$$3. (1) \text{ As } \omega = 2\pi f, 2\pi f = 314, \text{ i.e., } f = \frac{314}{2 \times \pi} = 50 \text{ Hz}$$

For Problems 4–5

4. (4) 5. (1)

4. (4) Equation of line AB is given by $y = 10 + \left(\frac{10}{T}\right)t$

$$y_{\text{av}} = \frac{1}{T} \int_0^T y dt = \frac{1}{T} \int_0^T \left(10 + \frac{10}{T}t\right) dt = \frac{1}{T} \left[10t + \frac{5t^2}{T}\right]_0^T = 15$$

5. (1) Mean square value

$$= \frac{1}{T} \int_0^T y^2 dt = \int_0^T \left(10 + \frac{10}{T}t\right)^2 dt$$

$$= \frac{1}{T} \int_0^T \left(100 + \frac{100}{T^2}t^2 + \frac{200}{T}t\right) dt$$

$$= \frac{1}{T} \left[100t + \frac{100t^3}{3T^2} + \frac{100t^2}{T}\right]_0^T = \frac{700}{3}$$

$$\text{rms value} = 10\sqrt{7/3}$$

For Problems 6–7

6. (3) 7. (1)

6. (3) When dc is applied, $I = \frac{V}{R}$, i.e., $R = \frac{12}{4} = 3 \Omega$

and when ac is applied, $Z = \left(\frac{V}{I}\right) = \left(\frac{12}{2.4}\right) = 5 \Omega$

$$R^2 + X_L^2 = 5^2 \quad (\text{as } Z = \sqrt{R^2 + X_L^2})$$

$$\text{So, } X_L^2 = 5^2 - R^2 = 5^2 - 3^2 = 4^2, \text{ i.e., } X_L = 4 \Omega$$

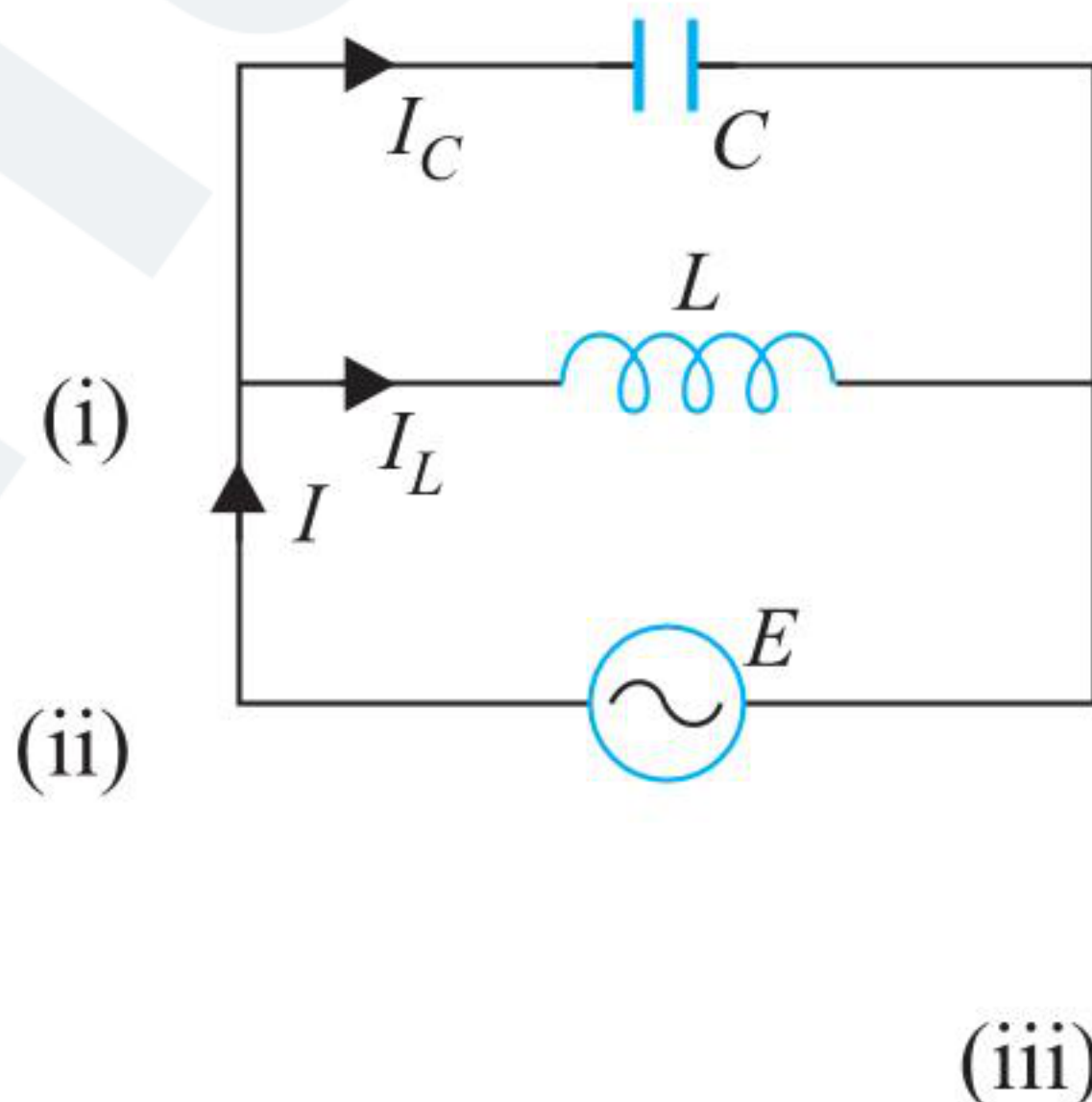
$$\text{but as, } X_L = \omega L, L = \frac{X_L}{\omega} = \frac{4}{50} = 0.08 \text{ H}$$

7. (1) Now when the capacitor is connected to the above circuit in series,

$$X_C = \frac{1}{\omega C} = \frac{1}{50 \times 2500 \times 10^{-6}} = \frac{10^3}{125} = 8 \Omega$$

$$\text{so, } Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{3^2 + (4 - 8)^2} = 5 \Omega$$

$$\text{and hence, } I_{\text{rms}} = \frac{V}{Z} = \frac{12}{5} = 2.4 \text{ A}$$



$$\text{So, } P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos \phi = (I_{\text{rms}} \times Z) \times I_{\text{rms}} \times \left(\frac{R}{Z} \right)$$

$$\text{i.e., } P_{\text{av}} = I_{\text{rms}}^2 R = (2.4)^2 \times 3 = 17.28 \text{ W}$$

For Problems 8–9

8. (1) 9. (1)

$$Z^2 = (X_C - X_L)^2 + R^2 = (31.85 - 6.28)^2 + (50)^2 = 3154$$

$$P = \left(\frac{E_{\text{rms}}^2}{Z^2} \right) R = \frac{(10/\sqrt{2})^2}{3154} \times 50 = 0.8 \text{ W}$$

$$\text{Heat produced in 20 min} = (0.8)(20 \times 60) = 960 \text{ J}$$

$$X_C - X_L = 31.85 - 2(6.28) = 19.29$$

$$I_m = \frac{10}{19.29} = 0.52$$

$$\text{Hence, } I = 0.52 \sin(314t + \pi/2) = 0.52 \cos 314t$$

For Problems 10–13

10. (1) 11. (4) 12. (2) 13. (4)

10. (1) As this circuit is a series LCR circuit, current will be maximum at resonance, i.e.,

$$\omega = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(4.9 \times 10^{-3})(10^{-6})}} = \frac{10^5}{7} \text{ rad s}^{-1}$$

$$\text{with } I = \frac{V}{R} = \frac{10}{(32 + 68)} = \frac{1}{10} \text{ A}$$

$$\text{So the impedance, } Z_p = [R_1^2 + (1/\omega C)^2]^{1/2} = \sqrt{5924} = 77 \Omega$$

$$11. (4) Z_Q = [R_2^2 + (\omega L)^2]^{1/2}$$

$$= [(68)^2 + (4.9 \times 10^{-3} \times 10^5/7)^2]^{1/2} = \sqrt{9524} = 97.6 \Omega$$

$$12. (2) V_p = IZ_p = \frac{1}{10} \times (77) = 7.7 \text{ V}$$

$$13. (4) V_Q = IZ_Q = \frac{1}{10} \times (97.6) = 9.76 \text{ V}$$

For Problems 14–16

14. (2) 15. (1) 16. (3)

$$\text{At resonance as } X = 0, I = \frac{V}{R} = \frac{60}{120} = \frac{1}{2} \text{ A}$$

$$\text{as } V_L = IX_L = I\omega L, L = \frac{V_L}{I\omega}$$

$$\text{So, } L = \frac{40}{(1/2) \times 4 \times 10^5} = 0.2 \text{ mH}$$

$$\omega = \frac{1}{\sqrt{LC}} \Rightarrow C = \frac{1}{L\omega^2}$$

$$\text{i.e., } C = \frac{1}{0.2 \times 10^{-3} \times (4 \times 10^5)^2} = \frac{1}{32} \mu\text{F}$$

Now in case of series LCR circuit,

$$\tan \phi = \frac{X_L - X_C}{R}$$

So current will lag the applied voltage by 45° if,

$$\tan 45^\circ = \frac{\omega L - \frac{1}{\omega C}}{R}$$

$$1 \times 120 = \omega \times 2 \times 10^{-4} - \frac{1}{\omega(1/32) \times 10^{-6}}$$

$$\omega^2 - 6 \times 10^5 \omega - 16 \times 10^{10} = 0$$

$$\text{i.e., } \omega = \frac{6 \times 10^5 \pm \sqrt{(6 \times 10^5)^2 + 64 \times 10^{10}}}{2}$$

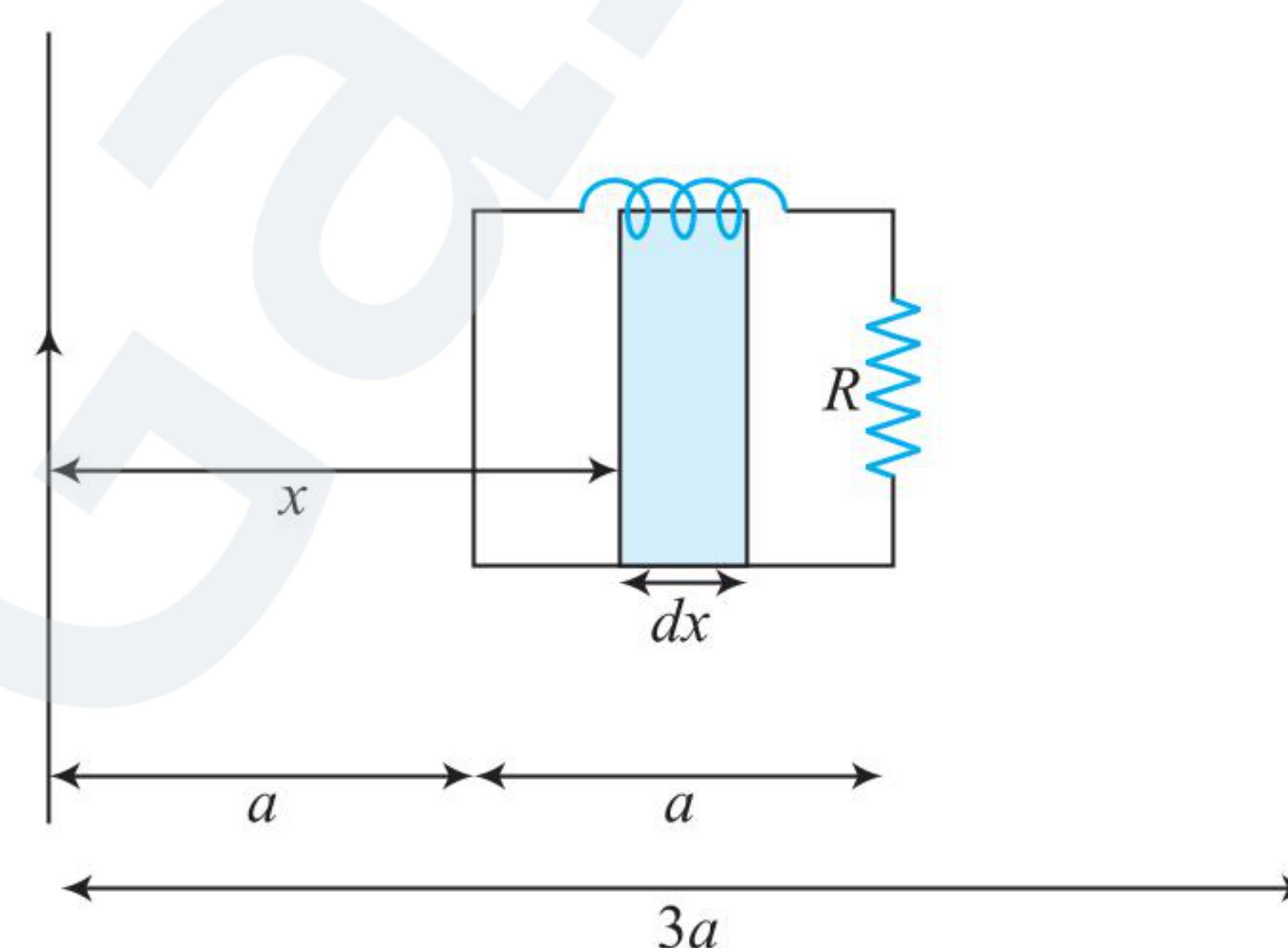
$$\text{i.e., } \omega = \frac{6 \times 10^5 + 10 \times 10^5}{2} = 8 \times 10^5 \text{ rad s}^{-1}$$

For Problems 17–19

17. (1) 18. (1) 19. (4)

$$d\phi = Bda$$

$$d\phi = \left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(3a-x)} \right] adx$$



$$\phi = \frac{\mu_0 I}{2\pi} \left[\int_a^{2a} \frac{dx}{x} + \int_a^{2a} \frac{dx}{(3a-x)} \right] a; \quad \phi = \frac{\mu_0 I a}{\pi} \ln 2$$

Magnitude of emf in this circuit:

$$\begin{aligned} \varepsilon &= \left| \frac{d\phi}{dt} \right| = \frac{\mu_0 a (\ln 2)}{\pi} \left| \frac{dI}{dt} \right| \\ &= \frac{\mu_0 a \ln 2}{\pi} I_0 \omega \sin \omega t \end{aligned}$$

$$\text{ac current, } I = \frac{\mu_0 a \ln 2 I_0 \omega}{\pi \sqrt{R^2 + \omega^2 L^2}} \sin(\omega t - \phi)$$

Matrix Match Type

1. i. → b., c.; ii. → b., c.; iii. → a., b., c., d.; iv. → b., c., d.

$$\text{i. } L = \frac{\mu N^2 A}{l}$$

So, L depends upon shape, size (l, A) and medium (μ) inserted.

$$\text{ii. } C = \frac{\varepsilon A}{d}$$

So, C depends upon shape, size (A, d) and medium (ε) inserted.

$$\text{iii. } Z = \sqrt{R^2 + x^2}$$

When $x_L = \omega L$, ω depends upon the external voltage source. So, Z depends upon all the factors in column II.

$$\text{iv. } X_C = \frac{1}{\omega C} \text{ (independent of resistivity)}$$

2. i. → d.; ii. → a., d.; iii. → b., d.; iv. → b.

i. For sinusoidal curve $i_{\text{rms}} = i_0/\sqrt{2}$ and average current for positive half cycle is $2i_0/\pi$.

ii. Let us find the average current (or rms current) for $t = 0$ to $T/4$. Then it will also be average current (or rms current) for half cycle, i.e., from $t = 0$ to $T/2$.

For $t = 0$ to $T/4$: $i = \frac{4i_0}{T}t$

$$\text{then } i_{av} = \frac{\int_0^{T/4} i dt}{T/4} = \frac{\int_0^{T/4} \frac{4i_0}{T} t dt}{T/4} = \frac{i_0}{2}$$

$$\text{and } i_{rms} = \sqrt{\frac{\int_0^{T/4} i^2 dt}{T/4}} = \sqrt{\frac{64i_0^2}{T^3} \int_0^{T/4} t^2 dt} = \frac{i_0}{\sqrt{3}}$$

Full cycle average current is zero.

iii. For positive half cycle, average current

$$= \frac{\int i dt}{\int dt} = \frac{i_0(T/2)}{T/2} = i_0$$

Full cycle average current is zero.

iv. Average current for positive half cycle is i_0 .

For full cycle, average current

$$= \frac{\int i dt}{\int dt} = \frac{i_0(T/2) + 0}{T} = \frac{i_0}{2}$$

$$\text{and } i_{rms} = \sqrt{\frac{\int_0^{T/2} i_0^2 dt}{T}} = \frac{i_0}{\sqrt{2}}$$

3. i. $\rightarrow$ b., c.; ii. $\rightarrow$ a., d.; iii. $\rightarrow$ a., c.; iv. $\rightarrow$ b., d.

$$\text{i. } \tan \phi = \frac{1/\omega C}{R} = 1 \Rightarrow \phi = \frac{\pi}{4}, \text{ current leads source voltage}$$

because reactance is capacitive.

$$\text{ii. Pure inductive circuit } \phi = \frac{\pi}{2}, \text{ current lags behind source}$$

voltage because reactance is inductive.

$$\text{iii. as } R = 0, \tan \phi = \infty (X_C > X_L) \\ \phi = \pi/2, \text{ current leads source voltage because reactance is capacitive.}$$

$$\text{iv. } \tan \phi = \frac{\omega L}{R} = 1 \Rightarrow \phi = \frac{\pi}{4}, \text{ current lags behind source}$$

voltage because reactance is inductive.

4. i. $\rightarrow$ b., d.; ii. $\rightarrow$ a.; iii. $\rightarrow$ a.; iv. $\rightarrow$ b., c.

$$X_L = \omega L = 2\pi \times 50 \times \frac{100}{\pi} \times 10^{-3} = 10 \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times \frac{100}{\pi} \times 10^{-6}} \Omega = 100 \Omega$$

Impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = 134.5 \Omega$$

rms value of the current through the circuit is

$$I_{rms} = \frac{220}{Z} = 1.63 \text{ A}$$

rms value of voltage drop across the capacitor is

$$V_{Cv} = I_{rms} X_C = 163 \text{ V}$$

Average power dissipated in the resistor is

$$P_{av} = I_{rms}^2 R = 1.63^2 \times 100 = 265.7 \text{ W}$$

Average power dissipated in inductor and capacitor would be zero.

5. i. $\rightarrow$ a., b.; ii. $\rightarrow$ c.; iii. $\rightarrow$ a., b., c., d.; iv. $\rightarrow$ d.

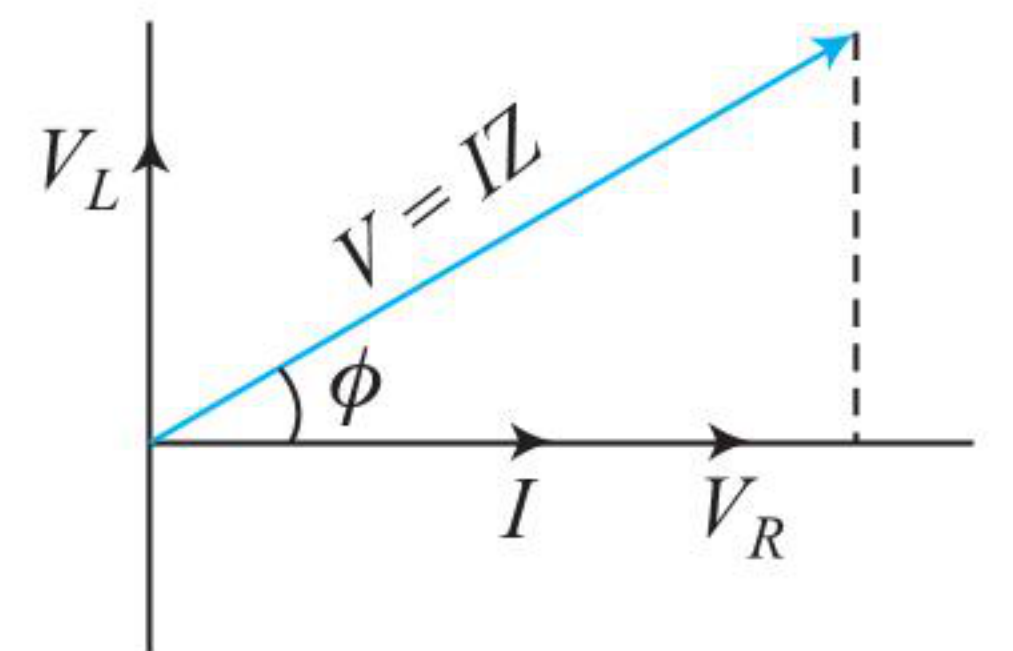
i. For LR series circuit, the phasor diagram is as shown below
 I lags voltage by an angle $\phi (< \pi/2)$.

I lags V_L by an angle $\pi/2$.

ii. For RC series ac circuit, I leads V by an angle less than $\pi/2$.

iii. Depending on the value of L , C and R , circuit would be either capacitive, inductive or purely resistive. All possibilities can be there.

iv. In purely resistive circuit, current and voltage are in same phase.



Numerical Value Type

1. a. (4), b. (1) In such cases it is difficult to develop a single equation. Hence, it is usual to consider two equations, one applicable from 0 to 1 ms and another from 1 to 2 ms.

For t lying between 0 and 1 ms, $V_1 = 4$. For t lying between 1 and 2 ms, $V_2 = -4t + 4$.

$$\text{(a) } V_{rms} = \sqrt{\frac{1}{2} \left(\int_0^1 V_1^2 dt + \int_1^2 V_2^2 dt \right)}$$

$$V_{rms}^2 = \frac{1}{2} \left[\int_0^1 4^2 dt + \int_1^2 (-4t + 4)^2 dt \right]$$

$$= \frac{1}{2} \left[16t \Big|_0^1 + \left| \frac{16t^3}{3} \right|_1^2 + 16t \Big|_1^2 - \left| \frac{32t^2}{2} \right|_1^2 \right] = \frac{32}{3}$$

$$V_{rms} = \sqrt{\frac{32}{3}} = 4\sqrt{\frac{2}{3}} \text{ V, Hence } N = 4$$

$$\text{(b) } V_{av} = \frac{1}{2} \left[\int_0^1 V_1 dt + \int_1^2 V_2 dt \right] = \frac{1}{2} \left[\int_0^1 4 dt + \int_1^2 (-4t + 4) dt \right]$$

$$= \frac{1}{2} \left[4t \Big|_0^1 + \left[-\frac{4t^2}{2} + 4t \right]_1^2 \right] = 1 \text{ V}$$

2. (5) From the $i - t$ graph (figure), area from $t = 0$ to $t = 2$ s

$$\frac{1}{2} \times 2 \times 10 = 10 \text{ A s}$$

$$\therefore \text{Average current} = \frac{10}{2} = 5 \text{ A}$$

$$\text{3. (0) } \langle i \rangle = \frac{\int_{\pi/2\omega}^{3\pi/2\omega} I_m \sin \omega t dt}{\frac{3\pi}{2\omega} - \frac{\pi}{2\omega}} = \frac{I_m \left(-\frac{\cos \omega t}{\omega} \right)_{\pi/2\omega}^{3\pi/2\omega}}{\frac{\pi}{\omega}} = 0$$

4. (5) Net current: $I = 3 + 4\sqrt{2} \sin \omega t$

$$\text{Now find } I_v = \sqrt{\frac{\int_0^T (3 + 4\sqrt{2} \sin \omega t)^2 dt}{T}} = 5 \text{ A}$$

$$\text{5. (8) } P_{av} = E_v I_v \cos \phi = E_v \frac{E_v}{Z} (R/Z)$$

Z has to be same in both cases.

$$Z_1 = Z_2 \Rightarrow \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (X_C - X_L)^2}$$

$$\Rightarrow X_L = X_C - X_L \Rightarrow 2X_L = X_C \Rightarrow 2\omega L = \frac{1}{\omega C}$$

$$\Rightarrow C = \frac{1}{2\omega^2 L}$$

$$\Rightarrow C = \frac{1}{2 \times (2\pi f)^2 L} = \frac{1}{2 \times 4\pi^2 \times (50)^2 \times 5/8} = 8 \times 10^{-6} \text{ F}$$

6. (4) $V_C - V_L = 120$

$$V_R^2 + (V_C - V_L)^2 = E_v^2$$

$$V_R^2 + (120)^2 = (200)^2 \Rightarrow V_R = 160$$

$$I = \frac{V_R}{R} = \frac{160}{40} = 4 \text{ A}$$

Archives

JEE Advanced

Single Correct Answer Type

1. (2) Impedance $Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2}$

As ω increases, Z decreases. Hence, bulb will glow brighter.

Multiple Correct Answers Type

1. (2), (3)

Case (A): $Z_A = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Case (B): $Z_B = \sqrt{R^2 + \left(\frac{1}{\omega KC}\right)^2}$

So $Z_B < Z_A$

$$I_R^A = \frac{V}{Z_A} \text{ and } I_R^B = \frac{V}{Z_B} \text{ clearly } I_R^A < I_R^B$$

Since current in case (B) is greater, so p.d. across R will increase in case (B) and thus across capacitor will decrease.

Hence $V_C^A > V_C^B$ ($\because V_R^2 + V_C^2 = V_0^2$)

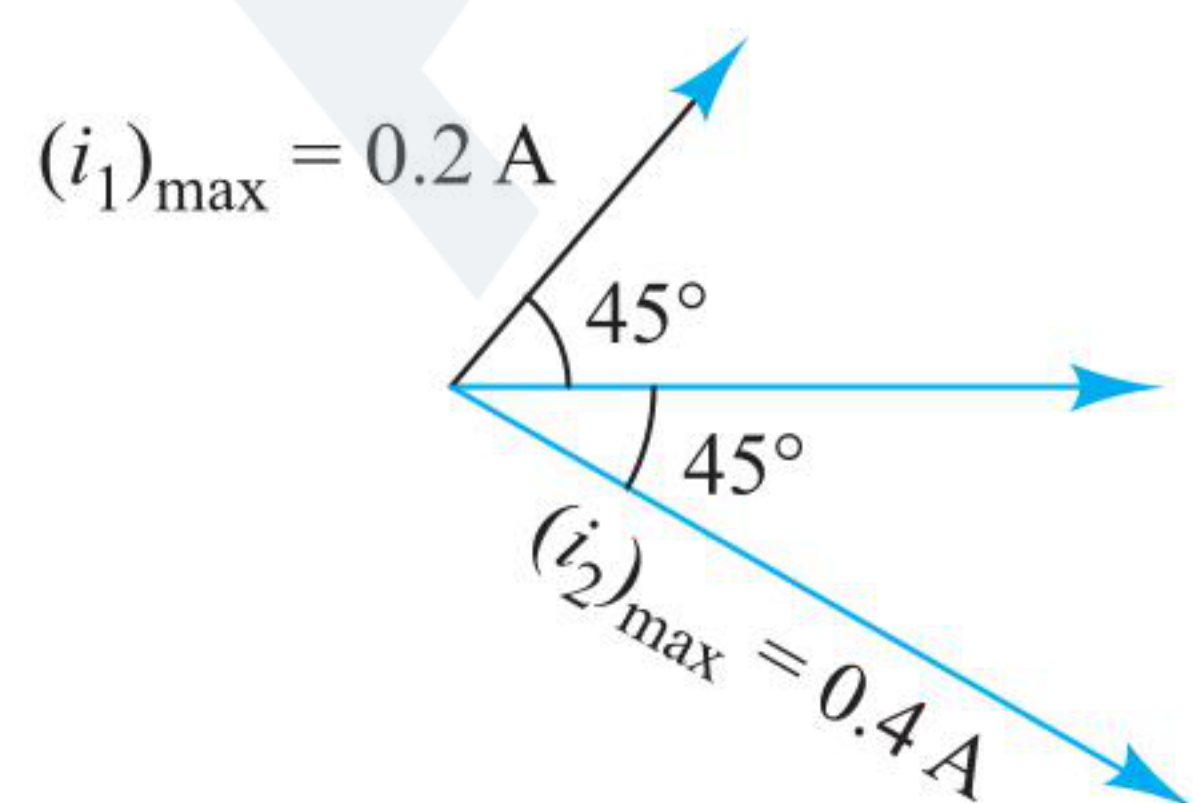
2. (1), (3)

$$C = 100 \mu\text{F}, \frac{1}{\omega C} = \frac{1}{(100)(100 \times 10^{-6})}$$

$$X_L = 100 \Omega, X_L = \omega L = (100)(0.5) = 50 \Omega$$

$$Z_1 = \sqrt{X_L^2 + 100^2} = 100\sqrt{2} \Omega$$

$$Z_2 = \sqrt{X_L^2 + 50^2} = \sqrt{50^2 + 50^2} = 50\sqrt{2}$$



$$\varepsilon = 20\sqrt{2} \sin \omega t$$

$$i_1 = \frac{1}{5} \sin\left(\omega t + \frac{\pi}{4}\right)$$

$$i_2 = \frac{20\sqrt{2}}{50\sqrt{2}} \sin\left(\omega t - \frac{\pi}{4}\right) = \frac{2}{5} \sin\left(\omega t - \frac{\pi}{4}\right)$$

$$i = \sqrt{(0.2)^2 + (0.4)^2} = (0.2) \sqrt{1+4} = \frac{1}{5} \sqrt{5} = \frac{1}{\sqrt{5}}$$

$$(I)_{\text{rms}} = \frac{1}{\sqrt{2}\sqrt{5}} = \frac{1}{\sqrt{10}} = \frac{\sqrt{10}}{10} \approx 0.3 \text{ A}$$

$$(V_{100\Omega}) = (I_1)_{\text{rms}} \times 100 = \left(\frac{0.2}{\sqrt{2}}\right) \times 100 = 10\sqrt{2} \text{ V}$$

$$(V_{50\Omega})_{\text{rms}} = \left(\frac{0.4}{\sqrt{2}}\right) \times 50 = \frac{20}{\sqrt{2}} = 10\sqrt{2} \text{ V}$$

3. (3), (4)

If q represents the charge on capacitor's upper plate:

$$I(t) = I_0 \cos(\omega t) = \frac{dq}{dt} \Rightarrow q(t) = \frac{I_0}{\omega} \sin(\omega t)$$

$$\text{Max charge} = \frac{1 \text{ A}}{500 \text{ rad s}^{-1}} = 2 \times 10^{-3} \text{ C}$$

$$\text{Charge on upper plate at } t = \frac{7\pi}{6\omega} = \frac{I_0}{\omega} \sin\left(\frac{7\pi}{6}\right) = -\frac{I_0}{2\omega}$$

When capacitor is fully charged, charge on upper plate
 $= 50 \text{ V} \times 20 \mu\text{F} = 1 \times 10^{-3} \text{ C}$

$$\therefore Q = 1 \times 10^{-2} \text{ C} - \left(-\frac{I_0}{2\omega}\right) = 2 \times 10^{-3} \text{ C}$$

Voltage across capacitor when A and D are connected

$$= \frac{1 \times 10^{-3} \text{ C}}{20 \times 10^{-6} \mu\text{F}} = 50 \text{ V}$$

Total voltage across resistor = 100 V

$$\Rightarrow \text{current} = \frac{100 \text{ V}}{10 \Omega} = 10 \text{ A}$$

Current is anticlockwise.

4. (1), (3)

We know $X_L = \omega L$ and $X_C = \frac{1}{\omega C}$

At $\omega \sim 0$,

$$X_L \sim 0 \text{ and } X_C \rightarrow \infty$$

$\therefore$ Current will be nearly zero.

$$\text{For } \omega = 10^6, X_L = \omega L = 10^6 \times 10^{-6} = 1 \Omega$$

$$\text{and } X_C = \frac{1}{\omega C} = \frac{1}{10^6 \times 10^{-6}} = 1 \Omega$$

For $\omega \gg 10^6$, $X_L \gg 1 \Omega$ and $X_C \ll 1 \Omega$ or $X_C \rightarrow 0$

Hence circuit does not behave like a capacitor.

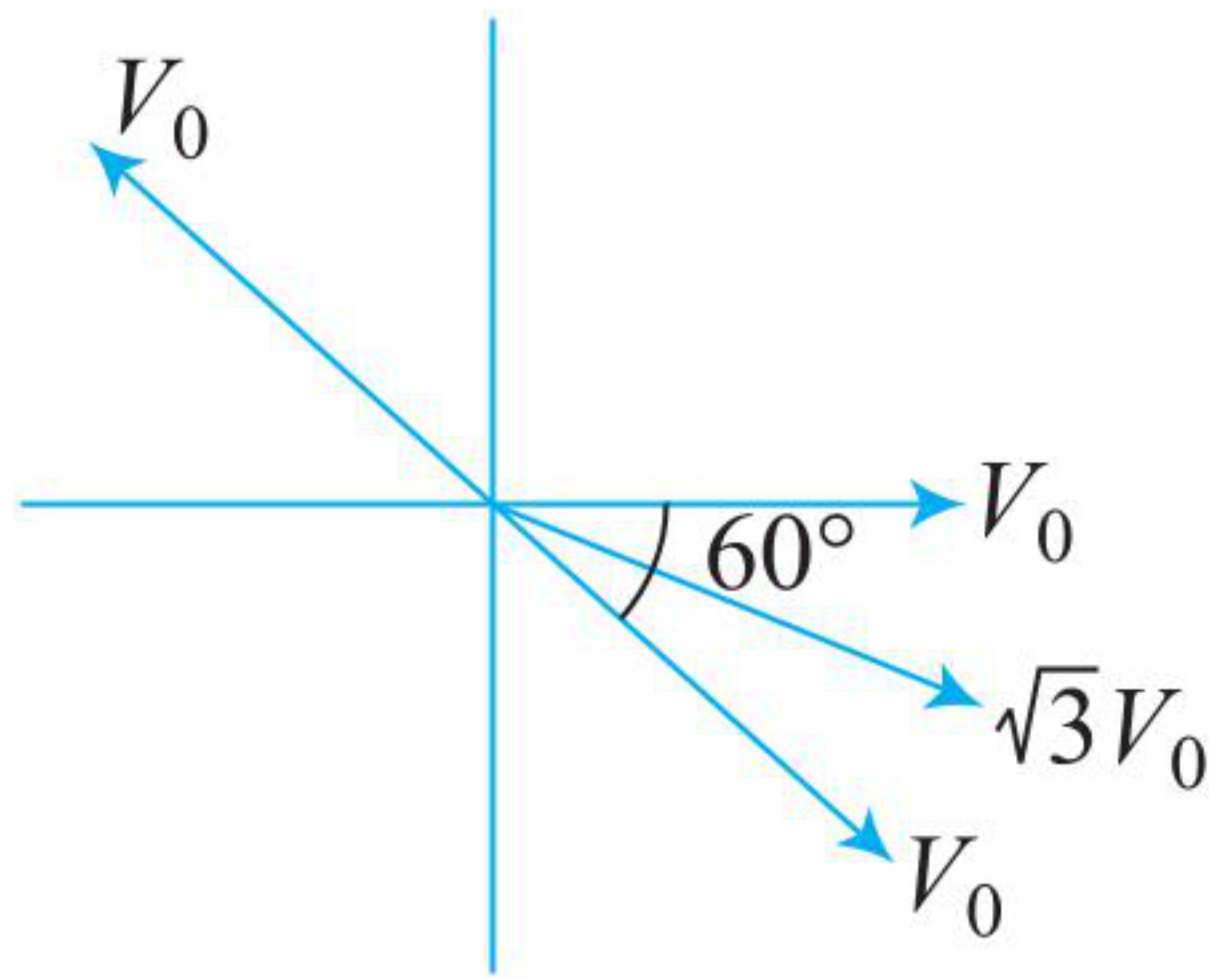
At resonance frequency (ω_R) i.e. when $X_C = X_L$ current will be in phase with voltage and frequency is independent of R .

Resonant frequency

$$\omega_R = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{10^{-6} 10^{-6}}} = 10^6 \text{ rad s}^{-1}$$

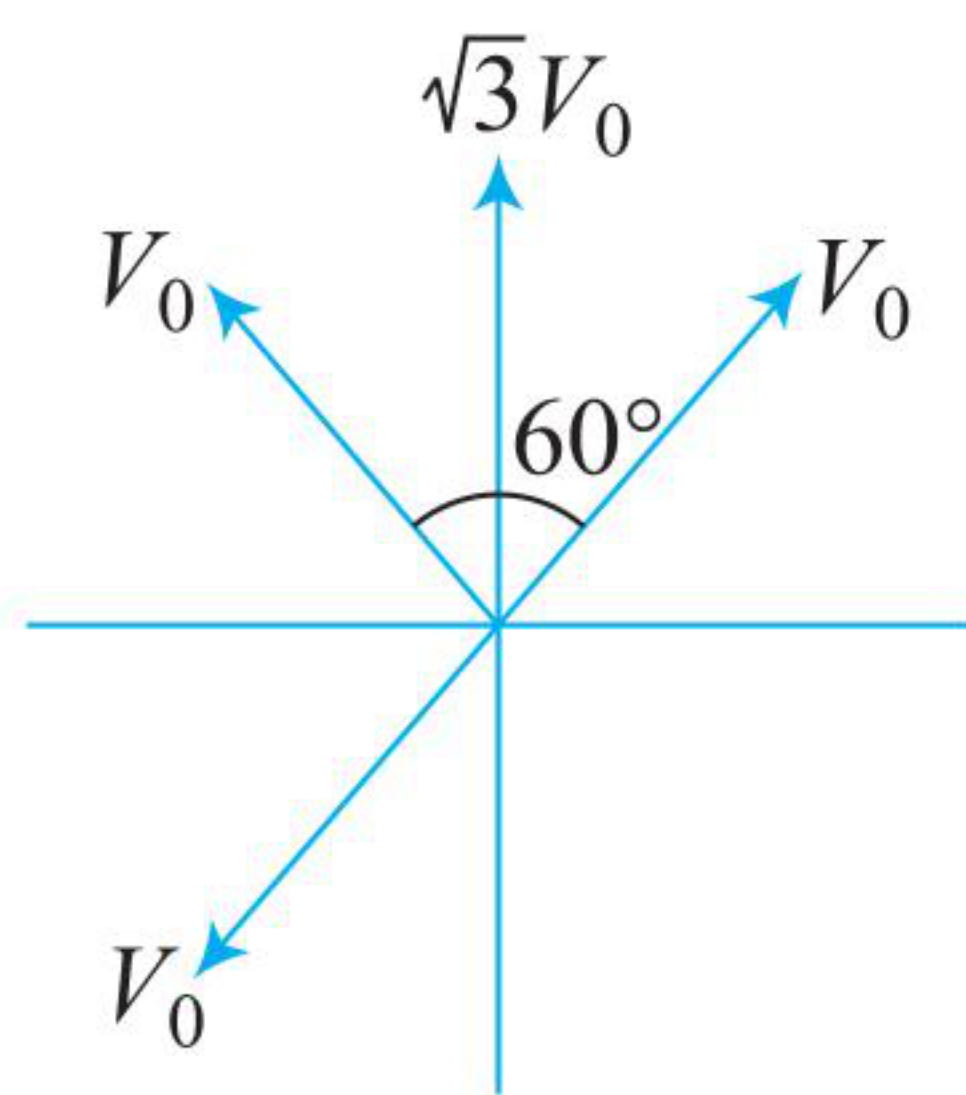
5. (3), (4)

$$V_{XY} = V_0 \sin \omega t - V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$$



$$V_{XY}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$$

$$V_{YZ} = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right) - V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$$



$$V_{YZ}^{rms} = \frac{\sqrt{3}V_0}{\sqrt{2}}$$

$\therefore$ Options (3) and (4) are correct.

Matrix Match Type

1. i. $\rightarrow$ c., d., e.; ii. $\rightarrow$ b., c., d., e.; iii. $\rightarrow$ a., b.; iv. $\rightarrow$ b., c., d., e.

a. In this case, steady state current is zero ($I = 0$) because of the presence of capacitor. Entire potential will be across capacitor in steady state. Hence $V_1 = 0$ and $V_2 = V$.

b. In steady state $V_1 = 0$, so $V_2 = 0$. Also V_2 is proportional to I .

c. $X_L = \omega L = 2\pi fL = 2\pi \times 50 \times 6 \times 10^{-3} = 1.885 \Omega$

$$R = 2 \Omega$$

Since $R > X_L$, so $V_2 > V_1$

Here $I \neq 0$

V_1 and V_2 both are proportional to I .

d. $X_L = 1.885 \Omega$,

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 3 \times 10^{-6}} = 1061 \Omega$$

Hence $X_C > X_L$, so $V_2 > V_1$

e. $X_C = 1.061 \text{ k}\Omega$, $R = 1 \text{ k}\Omega$

$X_C > R$, so $V_2 > V_1$

Numerical Value Type

1. (4) $\omega = 500 \text{ rad s}^{-1}$

$$Z = \sqrt{\left(\frac{1}{\omega C}\right)^2 + R^2} = R\sqrt{1.25}$$

$$\left(\frac{1}{\omega C}\right)^2 + R^2 = R^2(1.25) \Rightarrow \left(\frac{1}{\omega C}\right)^2 = \frac{R^2}{4}$$

$$\Rightarrow CR = \frac{2}{\omega} = \frac{2}{500} \text{ s} = \frac{2}{500} \times 10^3 \text{ ms} = 4 \text{ ms}$$

2. (100.00)

3. (60.00)

Considering the circuit diagram,

$$\sqrt{V_C^2 + V_R^2} = \epsilon_{rms}$$

$$\Rightarrow V_C^2 + 100^2 = 200^2$$

$$V_C = 100\sqrt{3} \text{ V}$$

$$\tan \phi = \frac{V_C}{V_R} = \frac{100\sqrt{3}}{100} \Rightarrow \phi = 60^\circ$$

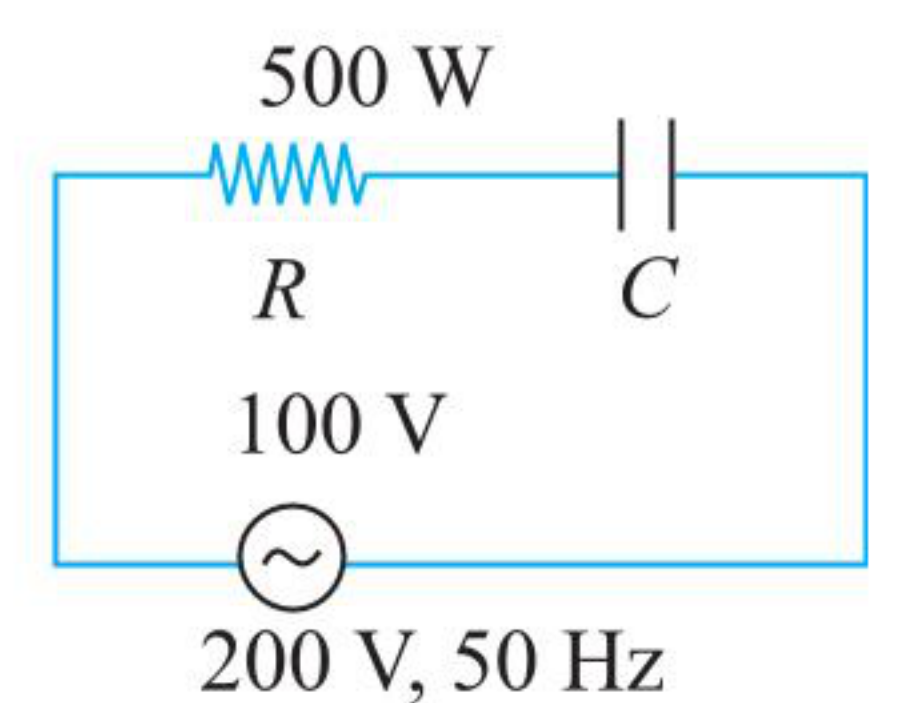
$$P = I_{rms} \epsilon_{rms} \cos \phi = \frac{\epsilon_{rms}^2}{Z} \cos \phi$$

$$500 = \frac{200}{Z} \frac{1}{2} \Rightarrow Z = 40 \Omega$$

$$\cos \phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{40} \Rightarrow R = 20 \Omega$$

$$\text{and } X_C = \sqrt{Z^2 - R^2} = \sqrt{40^2 - 20^2} = 20\sqrt{3} \Omega$$

$$\Rightarrow \frac{1}{\omega C} = 20\sqrt{3} \therefore C = 100 \text{ F}$$



...(i)

...(ii)

...(iii)